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A Multi-Objective Mathematical Model for Seating Plans in Public Places

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ABSTRACT

Social distance-based allocation models have gained importance since the advent of COVID-19 to minimize infection risk while continuing daily life. A multi-objective mathematical model with three objectives has been developed in this context. The proposed model considers various factors such as immune system, mask or vaccination status or air purifiers in the place to maximize the number of people while minimizing the total and maximum individual infection risk. The AUGMECON2 method is applied to the model and the model is applied in a real place. The results are analyzed using different parameters according to the immune system, mask or vaccination status of the allocated people. It has been observed that changing the parameters provides improvements of up to 45% in total and maximum individual infection risk. Thanks to Pareto sets obtained by the model, the decision-makers can make quick decisions between improved results.

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1. Introduction

The introduction of the concept of 'social distancing' has brought distance-based allocation planning in public places. Although widely discussed in the literature, the limited number of mathematical models incorporating social distancing has been a key source of inspiration for this study. Social distancing, often called physical separation, is a group of non-pharmaceutical measures intended to lessen the spread of infectious diseases by direct physical contact. Its primary objective is to keep people at a certain physical distance from one another to decrease the possibility of intimate contact and the danger of disease transmission that goes along with it (Moosa, 2020).

When allocating people according to social distancing guidelines, two objectives emerge: ensuring safety and maximizing the number of people. Achieving a balance between creating a safer place and allocating as many people as possible presents a trade-off (Fischetti et al., 2023). This is because increasing the number of people in a place raises the potential sources of infection. While the risk of infection increases with the number of people in the place, the impact of each person on the total infection risk varies. This variation is influenced by factors such as immune status, vaccination and mask usage (Huang et al., 2024), (Deka et al., 2024), (*WHO, Vaccination*, n.d.). The infection risk also affected by the air quality of the place (Lowther et al., 2023). People with strong immune systems, those vaccinated, and those who wear masks have a lower risk of infection. Additionally, the quality and cleanliness of the air also play a significant role in decreasing the risk of infection. Portable air purifiers are devices used in this direction (Lowther et al., 2023), (Cooper et al., 2021).

This study aims to propose a multi-objective mathematical model for allocating people in a defined space considering social distancing, the individual's immune system conditions, vaccination status and mask usage to minimize infection risk. The rest of the paper is organized as follows: Section 2 conducts a general literature review on these types of allocation models. Section 3 represents the proposed method and the mathematical model. The findings of the applications are presented in Section 4 and the conclusion and discussion are presented in Section 5.

2. Literature Review

The COVID-19 pandemic, subordinated by the novel SARS-COV-2 virus has substantially impacted the quality of life worldwide. Different approaches to stop or minimize the spread of COVID-19 include vaccination and non-pharmaceutical interventions. These interventions such as social distancing, mask-wearing and avoiding close contact with others preventive measures are required for the effective virus transmission control for COVID-19 and also resulted in notable changes in lifestyles of lots of people (Mun et al., 2024).

Social distancing, also referred to as physical distancing, is done to keep people physically apart from each other and limit the frequency of their close interactions to stop the spread of infection. Numerous actions are taken in response to it, such as closing schools and universities, restricting travel, and quarantine. Reducing the likelihood of an uninfected person and infected person's contact to decrease the number of deaths. Additionally, constructing a settled sum of physical distance between individuals also decreases near contact (Moosa, 2020). Governments took measures to limit the number of contacts between their citizens (Van Schaik et al., 2024). These distancing methods brought new metrics to our lives such as 1 meter which is recommended by World Health Organization and 2 meters recommended if masks are not worn and the expected time to spend exceeds 15 minutes (Bartolucci et al., 2022).

As the significance of social distance became clearer, the body of research in this area has grown. Air transport is among the industries that COVID-19 is having a significant impact on. Government-imposed preventive measures as well as travelers' anxiety about contracting diseases demonstrated the strong link between viruses and air travel. Airplanes' transportation capability fell sharply to almost nothing in some areas. Social distancing is the recommended preventive measure for air travel to compensate for this sharp decline. Various strategies have been employed as social distancing to effectively implement the distance rule (Salari et al., 2022), (Milne et al., 2022),(Milne et al., 2020),(Milne et al., 2021), (Pavlik et al., 2021). Studies on the application of social distance in public transport sectors other than aviation have also been popular in the literature (Moore et al., 2021). Various research has been conducted in the literature with the goal of showing the ways in which social distance lowers the spread of viruses in addition to public transportation and air transportation. The literature has included social distance allocation models which include seating groups of friends and family together while observing social distance, grouping people who share characteristics and keeping them apart from other people and setting up dining tables in restaurants. Although mathematical models are frequently used in these studies, metaheuristic algorithms have also been used where mathematical models are inadequate. Additionally, simulation models were used to simulate the models in some studies. Table 1 represents a few studies from the year 2021-2024 to represent and work on the relationship between social distance and virus transmission.

Table 1. Literature review.

Authors	Methodology	Fundamental aim
(Pavlik et al., 2021)	Mathematical Modelling	Minimizing virus transmission risk on airplanes.
(Moore et al., 2021)	Mathematical Modelling	Assigning passengers to the seats according to social distancing.
(Contardo & Costa, 2022)	Mathematical Modelling	Finding the best places for tables are socially distant.
(Dundar & Karakose, 2023)	Heuristics	Using social distancing to minimize the risk of infection.
(Xu et al., 2023)	Mathematical Modelling	Ensuring different social distancing requirements between groups.
(Fischetti et al., 2023)	Mathematical Modelling	Locating facilities according to social distancing.
(Yoon et al., 2024)	Simulation	Investigating the different simulation models for measuring the transmission risk of infectious diseases in indoor places.
(Banerjee et al., 2024)	Simulation and heuristic	Arranging the facilities to maximize social distancing while distributing essential amenities to populations.

According to the study of Fischetti et al. (Fischetti et al., 2023), allocating people according to social distance has become a new dimension, and it turns out to be a challenge. The article has two approaches to allocating people: first, fitting as many people as possible in each area and allocating a fixed number of people in a given area while fitting them most safely. The authors discovered an analogy between the allocation of wind turbines and the allocation of people. Customers were placed in an area with previously determined coordinates through a single-objective mathematical model written using a function that represents the risk of infection. However, since accommodating more people is the article's aim, only one consequence is achieved, and the article only provides an example of one requirement. Since this single-objective approach may not be very suitable for real life, multi-objective approaches are needed. Also, real-life conditions frequently change and multi-objective approaches give practical results. Beyond the distance factor, various factors such as health status, vaccination status or mask usage of individuals, the presence of air purifiers in the place also influence infection risk in real life. There is a lack of mathematical models in the literature that considers these various factors, this study has been developed to address this gap.

3. Research methodology and the proposed method

The mathematical model developed in this study has three objectives. The discrete set V defined in the model is a set which includes all potential allocation points for people in the place. x_i is a binary variable and i must be always from the set V. If the model allocates someone at i points, x_i is 1, otherwise 0. The j indices that are used in the model also used to define allocation points. d_{ij} represents the distance between two people. D_{min} the minimum social distance defined by decision-makers. I_{ij} function represents the infection risk between people i and j . w_i represents a continuous variable equal to total infection risk that belongs to i. The first objective is to maximize the total number of people in the area. The second objective is to minimize the total infection risk and third objective is minimize the maximum individual infection risk between allocated people. y_t is a binary variable and t must be always from the discrete set A. If there is an air purifier in t point, y_t is 1, otherwise 0. The maximum and minimum number of people can be allocated in the place is defined in Eq. (4). The maximum and minimum number of air purifiers in the place is defined in Eq. (5). The social distance constraint is always ensured by the model thanks to Eq. (6). $airpur$ is a scalar variable that is decided by decision-makers as 0.005 and it is used for deciding the effect of each portable air purifier. $dair_t$ is a variable and it calculates the distance between the person at point i and the portable air purifier. ai_t is a function represents in Eq. (7) which changes according to closeness of the air to the person. If the air purifier is close to person, the infection risk decrease. $Riskmultiplier$ is a parameter that represents in Eq. (8) is defined according to different real-life situations. For example, if a person has a weak immune system, but he or she vaccinated for the illness and wears mask, the risk multiplier for this person is defined as 2. And the infection risk is multiplied by 2. Or, if the immune system of the person is strong, he or she is unvaccinated and masked, the risk for this person is defined as 1.10. And the infection risk is multiplied by 1.10. Eq. (9) and Eq. (10) are constraints used for calculating the total infection risk. Eq. (11) represents a function according to distance between two people. Eq. (12) represents the relation between random z variable and total infection risk that belongs to i^{th} person. The model firstly calculates the I_{ij}

for the ith person, then multiply this value by related risk multiplier for i[th] person. And then, the model subtracts the effect of the air purifier. The closer and more the air purifier, the lower the risk of infection.

$$\text{Maximize} \sum_{i\in V} x_i \tag{1}$$

$$\text{Minimize} \sum_{i\in V} w_i \tag{2}$$

$$\text{Minimize } z \tag{3}$$

Subject to

$$N_{\min} \le \sum_{i\in V} x_i \le N_{\max} \tag{4}$$

$$air_{\min} \le \sum_{t\in A} y_t \le air_{\max} \tag{5}$$

$$x_i + x_j \le 1, \quad [i,j]\in E_1 = \{[i,j]: i,j\in V,\ d_{ij} < D_{\min,}\ \ i<j\} \tag{6}$$

$$ai_t = \frac{airpur}{dair_t}, \quad \mathrm{t} \in \mathrm{A} \tag{7}$$

$$riskmultiplier_i = \begin{cases} \text{if immune system is weak, vaccinated, masked; 2} \\ \text{if immune system is moderate, vaccinated, masked; 1.5} \\ \text{if immune system is strong, vaccinated, masked; 1} \\ \text{if immune system is weak, unvaccinated, unmasked; 2.5} \\ \text{if immune system is moderate, unvaccinated, unmasked; 1.75} \\ \text{if immune system is strong, unvaccinated, unmasked; 1.25} \\ \text{if immune system is weak, vaccinated, unmasked; 2.25} \\ \text{if immune system is weak, unvaccinated, masked; 2.25} \\ \text{if immune system is moderate, vaccinated, unmasked; 1.60} \\ \text{if immune system is moderate, unvaccinated, masked; 1.60} \\ \text{if immune system is strong, vaccinated, unmasked; 1.10} \\ \text{if immune system is strong, unvaccinated, masked; 1.10} \end{cases} \tag{8}$$

$$\sum_{j\in V}\left(\left(I_{ij} * riskmultiplier_i\right) * x_j\right) - \sum_{i\in V}\left(\left(y(t) * ai_t\right) * x_i\right) \le w_i + M_i(1-x_i), \quad \forall i\in V \tag{9}$$

$$w_i = \left(\left(\sum_{j\in V} I_{ij} * riskmultiplier_i\right) * x_j - \sum_{i\in V}\left(\left(y(t) * ai_t\right) * x_i\right)\right)$$

$$w_i = \begin{cases} \text{if } \ x_i = 1; \ \sum_{j\in V} I_{ij} * riskmultiplier_i - \sum_{i\in V}\left(y(t) * ai_t\right) \\ \text{if } \ x_i = 0; \ \ 0 \end{cases} \tag{10}$$

$$I_{ij} = \frac{1}{d_{ij}^3} \quad \forall i,j\in V \tag{11}$$

$$w_i \le z, \quad \forall i\in V \tag{12}$$

$$x_i \in \{0,1\}, \ \forall i\in V \tag{13}$$

$$w_i \ge 0, \ \forall i\in V \tag{14}$$

$$y_t \in \{0,1\}, \ \forall t\in A \tag{15}$$

The method that is used to solve the multi-objective mathematical model is the advanced version of the Epsilon constraint method. This method, which is used when solving multi-objective models, is realized by turning the most crucial objective into an objective function and all other objectives into constraints. While writing the right-hand sides of the constraints, idle variables are used in each iteration. Thanks to this method, which gives the solution as a Pareto set, the decision-maker can select from this set and decide on the most appropriate result for his needs (Mavrotas & Florios, 2013). The new and only objective for this method for a classical maximization problem are given in Eq. (16) (Şimşek et al., 2022):

$$\text{Max}\left(f_1(x) + eps * \left(\frac{s_2}{r_2} + 10^{-1} x \frac{s_3}{r_3} + ... + 10^{-(p-2)} x \frac{s_p}{r_p}\right) \right) \tag{16}$$

4. Case study and findings

In this section the AUGMECON2 method is applied, and the proposed model is coded in GAMS environment. The set V is defined by the coordinates of a real classroom from a public university in Turkey. All applications are performed on Intel(R) Core(TM) i9-13900H CPU @ 2.60 GHz computer with 16 GB RAM and solved in GAMS 45.7.0 environment using CPLEX solver. The dataset has 84 points, and the social distance between two people is defined as 3 meters by decision-makers. Set A is represented by 7 potential allocation points for air purifiers in the same classroom. All people who will be allocated are assumed to have a weak immune system, vaccinated and masked. The risk multiplier for each person according to the model is 2. The results yield in 1 minute and 37 seconds. Table 2 shows the Pareto set for this application.

Table 2. Pareto set for the application.

Number of people that allocated	Total infection risk	Maximum individual infection risk
8	7.74641	1.22714
7	5.09117	1.03822
6	2.81165	0.53308
6	2.81201	0.53311
6	2.81289	0.53297
5	1.66632	0.41547
5	1.66635	0.41629
4	0.73982	0.21599

The trade-off between the number of people and the total infection risk can be seen from the Pareto set for the application. The decision-maker can decide which allocation is suitable for the place and can take that option to allocate their places. The allocation for 8 people in the classroom can be seen from Figure 1. Red points show all allocated people in the place, blue points show all potential allocation points for people. Purple points show all allocated air purifiers and yellow points show other potential allocation points for air purifiers.

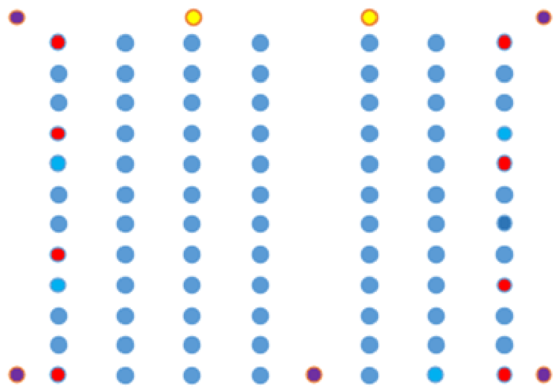

Fig. 1. Allocation in the classroom.

To compare the results and observe the differences in the scenario where more people are allocated, the dataset is expanded to 100 points to cover a larger area in the same classroom. The social distance between two people is defined as 3 meters. There are 7 potential allocation points for air purifiers in the same classroom. To compare the results, health status is defined with the first parameter 1 and parameter 12 and results are analyzed. Parameter 1, which represents a weak immune system, vaccinated and masked, is used first, and the model provides results in 26 minutes; results are shown in Table 3.

Table 3. Pareto Set for the Application (100 points-parameter 1).

Number of people that allocated	Total infection risk	Maximum individual infection risk
10	15.93574	2.08320
9	11.19644	1.64127
9	11.87190	1.55702
8	7.72813	1.12564
8	7.72792	1.12566
7	4.74516	0.87010
7	4.75578	0.86982
7	4.74636	0.86999
6	2.77294	0.60217
6	2.77691	0.60100
5	1.44920	0.38786
5	1.44543	0.38853
5	1.44500	0.38962

The parameter is changed to represent people with a strong immune system, unvaccinated, and masked. The model provides results in 27 minutes, and the results are shown in Table 4.

Table 4. Pareto Set for the Application (100 points-parameter 12).

Number of people that allocated	Total infection risk	Maximum individual infection risk
10	8.73215	1.14400
9	6.12755	0.89707
9	6.50310	0.85073
9	6.50423	0.85074
8	4.22541	0.61747
7	2.58505	0.47686
7	2.58332	0.47676
6	1.50248	0.32976
5	0.77744	0.21033
5	0.77635	0.21150
5	0.77592	0.21259

In the scenario where all people have weak immune system, everyone is vaccinated and masked, the total infection risk is 15.93574 and the maximum individual infection risk is 2.08320. In an allocation of 10 people where all people have strong immune system, everyone is unvaccinated but masked, the total infection risk is 8.73215 and the maximum individual infection risk is 1.14400. It has been observed that the immune system, vaccination, and mask status of the people change; there is an improvement of up to 45% in the results. Both models allocate maximum 10 people in the place but the Pareto sets have different options.

5. Conclusion and Discussion

Within the scope of allocating people to a place considering social distance and environmental factors in situations such as COVID-19 and similar outbreaks, rather than completely ending physical contact, the focus is on exploring the other ways in this study. Because complete physical isolation and quarantine are not always sustainable, optimizing the available place to minimize infection risk and keep people together while considering social distance can significantly contribute to overall public physical and mental health.

This study extends previous studies in the field of social distancing and mathematical modelling. While prior studies have primarily focused solely on social distancing, most have used various functions to calculate infection risk, often relying on microphysical models. Furthermore, while several studies have used agent-based simulations to analyze the spread of infection according to different social distance parameters, they often lack an optimization component that provides a fixed allocation according to people's features. Besides that, such approaches make it challenging to establish a general and fixed method for allocating individuals, particularly in scenarios where decision-makers have limited time to determine optimal allocation. Moreover, most studies aim to find a single optimal solution for allocating people to the place. However, in real-life scenarios, there is a trade-off between maximizing the number of people and minimizing the infection and decision-makers can make more balanced choices thanks to this. Providing Pareto optimal solutions quickly is one of the key strengths of this mathematical model and decision-makers can quickly adapt to changing conditions. With the help of the presented multi-objective mathematical model, any dataset and any parameters including people's health status, vaccination and mask usage, or the number of air purifiers in the allocated place can be used allocation people in a place while considering total and individual infection risk. Thus, a fair allocation is made and everyone who allocated is exposed to similar infection risk and no individual is disproportionately exposed to higher infection risk. Unlike static distancing measures, the proposed model allows a more flexible and adaptive decision support system for place owners who want to allocate more people safely in the place.

The multi-objective mathematical model developed in this study considers key factors that influence infection risk by using risk multiplier, including individuals' immune system, vaccination history and mask usage. Additionally, the number of air purifiers in the environment is used as a decision variable, allowing decision-makers to optimize both people allocation and air purifier allocation simultaneously. Although a classroom is used as a case study, the model is adaptable to various environments such as restaurants, cafes, offices, and other shared places while considering allocation and infection risk balance.

The computational complexity and solving time of the proposed method are limitations when using more data to cover larger places. As the number of potential allocation points and area of the place increases, solving the model may become impractical due to time considerations. In these situations, near optimal solutions can be investigated using meta-heuristic algorithms such as crow search algorithms, genetic algorithms or particle swarm optimization. In order to decrease solving time and finding applicable solutions in a reasonable time, future research would focus on integrating these algorithms.

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