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A new approach to solving fully fuzzy linear programming problems using the MOLP problem

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Abstract

This note shows that Definition 2.6 which is used by Ezzati et al. (2015) in failed to compare any arbitrary triangular fuzzy numbers. After that, Bhardwaj and Kumar (2015) based on this definition proposed an algorithm to convert a fully fuzzy programming problem with inequality constraints into a fully fuzzy linear programming problem with equality constraints and regarding the mentioned definition has concluded that the main problem was not an infeasible problem. We demonstrate that their presented method is not well in general, thus the proposed method to find the fuzzy optimal solution of fully fuzzy linear programming problems by Ezzati et al. (2013), can be improved by using some general definitions and a new version is provided in this note. An example is also presented to demonstrate the new form.

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1. Introduction

Fully Fuzzy Linear Programming (FFLP) problem in which all the parameters and variables are considered as fuzzy numbers is an attractive topic for researchers. (Lotfi et al., 2009; Kumar et al., 2011; Ezzati et al., 2013).

In Lotfi et al. method (2009), the parameters of fully fuzzy linear programming problem have been approximated to the nearest symmetric triangular fuzzy numbers. Then a fuzzy optimal approximation solution has been achieved by solving a multi-objective linear programming (MOLP) problem. In Kumar et al. (2011) method, the linear ranking function has been used to convert the fuzzy objective function to crisp objective function. Bhardwaj and Kumar (2015) showed that the fully fuzzy programming problems with inequality constraints cannot be transformed into fully fuzzy linear programming problems with equality constraints. And hence, the algorithm, proposed by Ezzati et al. for solving fully fuzzy linear programming problems with equality constraints, cannot be used for finding the fuzzy optimal solution of fully fuzzy linear programming problems with inequality constraints.

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Ezzati et al. (2015) introduced a definition to comparing triangular fuzzy numbers and using it. Proposed a new algorithm to find the optimal solution of Fully Fuzzy Linear Programming (FFLP) problem. Based on a new lexicographic ordering on triangular fuzzy numbers, a novel algorithm is proposed to solve the FFLP problem by converting it to its equivalent a Multi-Objective Linear Programming (MOLP) problem and then it is solved by the lexicographic method. Subsequently, other researchers have applied different ratings and definitions for comparing fuzzy numbers and solving a fully fuzzy linear programming problem (Lotfi et al., 2009; Naseri and Mahdavi-Amiri, 2009; Naseri et al., 2014; Naseri et al., 2017).

In this paper, we study Ezzati et al. (2015). By using of some definitions and numerical examples shown that Ezzati's definition for comparing triangular fuzzy numbers is not held for each fuzzy numbers. Examples are provided to prove this claim and by use of this fact propose a method to converting fully fuzzy problem into multi-objective linear programming problem, which is improved.

The rest of this paper is organized as follows: In Section 2, we review the basic definitions and results on fuzzy sets and some related topics. Section 3 gives the definition for comparing fuzzy numbers and then, we propose some numerical examples. Furthermore, we introduce a new method for solving FFLP problem in Section 3 and finally, the conclusions are discussed in Section 4.

2.Preliminaries

In this section, we begin with some basic definitions, arithmetic operations of fuzzy numbers and an existing ranking approach for comparing fuzzy numbers will be used in the rest of the paper.

Definition 1: Let R denote a universal set. Then, a fuzzy subset $\tilde{A}$ of R is defined by its membership function $\mu_{\tilde{A}}:\mathbb{R}\to[0,1]$; which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$.

To each element $x\in\mathbb{R}$, where the value of $\mu_{\tilde{A}}(x)$ at x shows that grade of membership of x in $\tilde{A}$.

A fuzzy subset $\tilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(R)$ and is often written $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)),x\in\mathbb{R}\}$; the class of fuzzy sets on $\mathbb{R}$ is denoted by $TF(\mathbb{R})$.

Definition 2: A fuzzy number $\tilde{A}=\langle x_1^l,y_1^c,z_1^u\rangle$ is said to be a triangular fuzzy number if its membership function is given as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-x_1}{y_1-x_1}, & x_1\le x\le y_1,\\ \dfrac{x-z_1}{y_1-z_1}, & y_1\le x\le z_1,\\ 0, & \text{Otherwise.}\end{cases} \tag{1}$$

Definition 3: A triangular fuzzy number $\tilde{A}=\langle x_1^l, y_1^c, z_1^u\rangle$ is said to be a non- negative triangular fuzzy number if and only if $x_1^l \ge 0$. The set of all these triangular fuzzy numbers is denoted by $TF(\mathbb{R})^+$.

Definition 4: Let $\tilde{A}=\langle x_1^l, y_1^c, z_1^u\rangle$ and $\tilde{B}=\langle x_2^l, y_2^c, z_2^u\rangle$ be two triangular fuzzy numbers. Then, arithmetic operation on these fuzzy numbers can be defined as follows:

(i)Addition: $\tilde{A}\oplus\tilde{B}=\left(x_1^l, y_1^c, z_1^u\right)+\left(x_2^l, y_2^c, z_2^u\right)=\left(x_1^l+x_2^l, y_1^c+y_2^c, z_1^u+z_2^u\right)$,

(ii) Subtraction: $\tilde{A}\Theta\tilde{B}=\left(x_1^l, y_1^c, z_1^u\right)-\left(x_2, y_2, z_2\right)=\left(x_1^l-x_2^l, y_1^c-y_2^c, z_1^u-z_2^u\right)$,

(iii) Multiplication: if $\tilde{B}$ be a non- negative triangular fuzzy number then:

$$\tilde{A}\otimes\tilde{B}=\begin{cases}\left(x_1^l x_2^l, y_1^c y_2^c, z_1^u z_2^u\right), & x_1^l\ge 0,\\ \left(x_1^l z_2^u, y_1^c y_2^c, z_1^u z_2^u\right), & x_1^l<0, z_1^u\ge 0,\\ \left(x_1^l z_2^u, y_1^c y_2^c, z_1^u x_2^l\right), & z_1^u<0.\end{cases} \tag{2}$$

Definition 5: An effective approach for ordering the elements of $\mathcal{F}(\mathbb{R})$ is to define a ranking function $\mathcal{R}:\mathcal{F}(\mathbb{R})\to\mathbb{R}$ which maps each fuzzy number into the real line, where a natural order exists.

We define orders on $\mathcal{F}(\mathbb{R})$ by

$\tilde{A}\ge_{\mathcal{R}}\tilde{B}$ *if and only if* $\mathcal{R}(\tilde{A})\ge\mathcal{R}(\tilde{B})$,

$\tilde{A}>_{\mathcal{R}}\tilde{B}$ *if and only if* $\mathcal{R}(\tilde{A})>\mathcal{R}(\tilde{B})$,

$\tilde{A}=_{\mathcal{R}}\tilde{B}$ *if and only if* $\mathcal{R}(\tilde{A})=\mathcal{R}(\tilde{B})$,

where $\tilde{A}$ *and* $\tilde{B}$ are in $\mathcal{F}(\mathbb{R})$.

As well as given in [5], we use ranking function for triangular fuzzy number $\tilde{A}=\langle x_1^l, y_1^c, z_1^u\rangle$ as : $\mathcal{R}(\tilde{A})=\frac{1}{4}\left(x_1^l+2y_1^c+z_1^u\right)$

Definition 6 : Two triangular fuzzy numbers $\tilde{A}=\langle x_1^l, y_1^c, z_1^u\rangle$ and $\tilde{B}=\langle x_2^l, y_2^c, z_2^u\rangle$ are said to be equal, $\tilde{A}=\tilde{B}$ if and only if $x_1^l=x_2^l$, $y_1^c=y_2^c$ and $z_1^u=z_2^u$.

3. Main results

Ezzati et al. (2015) presented a solution algorithm for all fully fuzzy problems with non-negative triangular fuzzy variables. Which this assumption limits the algorithm to solving specific problems and reduce the generality of the method to solve all fully fuzzy problems. Kumar pointed out an example that this method is not efficient to solve a fully fuzzy programming problems with inequality constraints. According to Definition 2.6 the optimal solution of the problem, obtained from Ezzati's method, is not a feasible fuzzy solution of the original problem. Ezzati et al. in Definition 2.6, proposed a method for comparing any arbitrary fuzzy numbers that by use of yager's ranking function in Definition 5 and solve the following counterexamples shown that their definition for all fuzzy numbers will not be right.

Counterexample: Consider two triangular fuzzy numbers $\tilde{A}=\langle 0,1,1.5\rangle$ and $\tilde{B}=\langle 0,1,1.6\rangle$ then according to (ii) of Ezzati's definition (Definition 2.6 of [1]) in this example we have that 1=1, $1.6>1.5$ then $\tilde{B}<\tilde{A}$ while due to the Definition 5 of this paper (Yagre's ranking function), $\mathcal{R}(\tilde{A})=\frac{1}{4}(0+2+1.5)=1.125$ and $\mathcal{R}(\tilde{B})=\frac{1}{4}(0+2+1.6)=1.115$ since $\mathcal{R}(\tilde{A})<\mathcal{R}(\tilde{B})$ then $\tilde{A}<\tilde{B}$. Mentioned explanations for the two above numbers and compare them more in Figure 1 clearly has been shown.

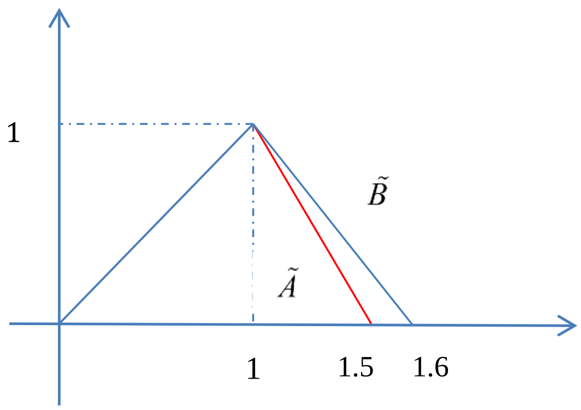

Fig. 1. Showing two triangular fuzzy numbers $\tilde{A}=\langle 0,1,1.5\rangle$ and $\tilde{B}=\langle 0,1,1.6\rangle$

Ezzati et al. (2015) represented an algorithm to solve each fully fuzzy problem based on a comparing method. We have shown that their comparing method and definition is not true for all fuzzy numbers and by disproving Ezzati's method. In this way, by use of Definition 4, we convert the objective function into three objectives as follows:

Consider the following Fully Fuzzy Linear Programming (FFLP) problem

$$\begin{aligned} &max\,(min)\quad \tilde{C}\otimes\tilde{X} \\ &s.t. \qquad\quad \tilde{A}\otimes\tilde{X}=\tilde{b} \end{aligned} \tag{3}$$

where $\tilde{X}$ is a non-negative triangular fuzzy number. In problem (3), we have that $\tilde{C} = \langle c^l, c^c, c^u \rangle$, $\tilde{X} = \langle x^l, x^c, x^u \rangle$, $\tilde{A} = \langle A^l, A^c, A^u \rangle$ and $\tilde{b} = \langle b^l, b^c, b^u \rangle$ with $x^l \geq 0$. By use of Definition 4 and with regard that $\tilde{X}$ is a non-negative triangular fuzzy number we can obtain $\tilde{C}^T \otimes \tilde{X} = \left\langle \left(c^T x\right)^l, \left(c^T x\right)^c, \left(c^T x\right)^u \right\rangle$, $\tilde{A} \otimes \tilde{X} = \left\langle (Ax)^l, (Ax)^c, (Ax)^u \right\rangle$.

With regard to Definition 6, problem (3) written as follows:

$$
\begin{aligned}
&max\,(min) \quad \tilde{C}^T \otimes \tilde{X} = \left\langle \left(c^T x\right)^l, \left(c^T x\right)^c, \left(c^T x\right)^u \right\rangle \\
&s.t. \quad (Ax)^l = b^l,\ (Ax)^C = b^C,\ (Ax)^u = b^u, \\
&\qquad x^c - x^l \geq 0,\ x^u - x^c \geq 0, x^l \geq 0.
\end{aligned} \tag{4}
$$

Now, by applying Definitions 4 and 6 we convert problem (4) into the MOLP problem with three objective functions as follow:

$$
\begin{aligned}
&max\,(min) \quad \left(c^T x\right)^C \\
&min\,(max) \quad \left(C^T x\right)^C - \left(C^T x\right)^l \\
&max\,(min) \quad \left(C^T x\right)^u - \left(C^T x\right)^C \\
&s.t. \quad (Ax)^l = b^l,\ (Ax)^C = b^C,\ (Ax)^u = b^u, \\
&\qquad x^c - x^l \geq 0,\ x^u - x^c \geq 0, x^l \geq 0,
\end{aligned} \tag{5}
$$

The lexicographic method will be used to obtain a lexicographically optimal solution of problem (5), so, we have:

$$
\begin{aligned}
&max\,(min) \quad \left(C^T x\right)^C \\
&s.t. \quad (Ax)^l = b^l,\ (Ax)^C = b^C,\ (Ax)^u = b^u, \\
&\qquad x^c - x^l \geq 0,\ x^u - x^c \geq 0, x^l \geq 0,
\end{aligned} \tag{6}
$$

If problem (6) has a unique optimal solution, namely $\tilde{X}^* = \left\langle \left(x^*\right)^l, \left(x^*\right)^c, \left(x^*\right)^u \right\rangle$, then it is an optimal solution of problem (4) and stop.

Otherwise, solve the following problem over the optimal solutions that are achieved in above as follow:

$$
\begin{aligned}
&min\,(max) \quad \left(c^T x\right)^C - \left(c^T x\right)^l \\
&s.t. \quad \left(c^T x\right)^C = m^*, \\
&\qquad (Ax)^l = b^l,\ (Ax)^C = b^C,\ (Ax)^u = b^u, \\
&\qquad x^c - x^l \geq 0,\ x^u - x^c \geq 0, x^l \geq 0,
\end{aligned} \tag{7}
$$

where m^* is the optimal value of problem (6).

If problem (7) has a unique optimal solution, namely $\tilde{X}^* = \left\langle \left(x^*\right)^l, \left(x^*\right)^c, \left(x^*\right)^u \right\rangle$, then it is also an optimal solution of problem (4) and stop.

Otherwise solve the following problem over the optimal solutions that are achieved in recently problem as follows:

$$\begin{aligned}
&max\ (min) && \left(c^T x\right)^u - \left(c^T x\right)^c \\
&s.t. && \left(C^T x\right)^C - \left(C^T x\right)^l = n^*, \\
& && \left(C^T x\right)^C = m^*, \qquad\qquad (8)\\
& && \left(Ax\right)^l = b^l,\ \left(Ax\right)^C = b^C,\ \left(Ax\right)^u = b^u, \\
& && x^c - x^l \geq 0, x^u - x^c \geq 0, x^l \geq 0,
\end{aligned}$$

where n^* is the optimal value of problem (8). So, the optimal solution of problem (4), namely $\tilde{X}^* = \left\langle \left(x^*\right)^l, \left(x^*\right)^c, \left(x^*\right)^u \right\rangle$ is obtained by solving problem (8).

Example3: Consider the following FFLP problem as follows:

$$\begin{aligned}
max\quad & \langle 10,15,17\rangle \otimes \langle x_1^l, x_1^c, x_1^u\rangle \oplus \langle 10,16,20\rangle \otimes \langle x_2^l, x_2^c, x_2^u\rangle \oplus \\
& \langle 10,14,17\rangle \otimes \langle x_3^l, x_3^c, x_3^u\rangle \oplus \langle 10,12,14\rangle \otimes \langle x_4^l, x_4^c, x_4^u\rangle \\
s.t.\quad & \langle 8,10,13\rangle \otimes \langle x_1^l, x_1^c, x_1^u\rangle \oplus \langle 10,11,13\rangle \otimes \langle x_2^l, x_2^c, x_2^u\rangle \oplus \\
& \langle 9,12,13\rangle \otimes \langle x_3^l, x_3^c, x_3^u\rangle \oplus \langle 11,15,17\rangle \otimes \langle x_4^l, x_4^c, x_4^u\rangle = \\
& \langle 271.75, 411.75, 573.75\rangle, \\
& \langle 12,14,16\rangle \otimes \langle x_1^l, x_1^c, x_1^u\rangle \oplus \langle 14,18,19\rangle \otimes \langle x_2^l, x_2^c, x_2^u\rangle \oplus \\
& \langle 14,17,20\rangle \otimes \langle x_3^l, x_3^c, x_3^u\rangle \oplus \langle 13,14,18\rangle \otimes \langle x_4^l, x_4^c, x_4^u\rangle = \\
& \langle 385.5, 539.5, 759.5\rangle,
\end{aligned}$$

where $x_j^c - x_j^l \geq 0$, $x_j^u - x_j^c \geq 0$, $x_j^l \geq 0$ for all $j = 1, 2, 3, 4$.

Now, we convert the objective function into three objective functions as follows:

$$
\begin{array}{ll}
max & \left(15x_1^c + 16x_2^c + 14x_3^c + 12x_4^c\right) \\
min & \left(15x_1^c + 16x_2^c + 14x_3^c + 12x_4^c\right) - \left(10x_1^l + 10x_2^l + 10x_3^l + 10x_4^l\right) \\
max & \left(17x_1^u + 20x_2^u + 17x_3^u + 14x_4^u\right) - \left(15x_1^c + 16x_2^c + 14x_3^c + 12x_4^c\right) \\
s.t. & 8x_1^l + 10x_2^l + 9x_3^l + 11x_4^l = 271.75, \\
 & 10x_1^c + 11x_2^c + 12x_3^c + 15x_4^c = 411.75, \\
 & 13x_1^u + 13x_2^u + 13x_3^u + 17x_4^u = 573.75, \\
 & 12x_1^l + 14x_2^l + 14x_3^l + 13x_4^l = 385.5, \\
 & 14x_1^c + 18x_2^c + 17x_3^c + 14x_4^c = 539.5, \\
 & 16x_1^u + 19x_2^u + 20x_3^u + 18x_4^u = 759.5, \\
 & x_j^c - x_j^l \geq 0,\ x_j^u - x_j^c \geq 0, x_j^l \geq 0 \ for\ all\ j = 1,2,3,4.
\end{array}
$$

The optimal solution of the above problem is achieved as follows:

$$
\tilde{X}^* = \begin{cases}
\tilde{x}_1^* = \left\langle \left(x_1^*\right)^l, \left(x_1^*\right)^C, \left(x_1^*\right)^u \right\rangle = \langle 17.28, 17.28, 17.28 \rangle, \\
\tilde{x}_2^* = \left\langle \left(x_2^*\right)^l, \left(x_2^*\right)^C, \left(x_2^*\right)^u \right\rangle = \langle 2.16, 2.16, 2.16 \rangle, \\
\tilde{x}_3^* = \left\langle \left(x_3^*\right)^l, \left(x_3^*\right)^C, \left(x_3^*\right)^u \right\rangle = \langle 4.65, 9.97, 16.37 \rangle, \\
\tilde{x}_4^* = \left\langle \left(x_4^*\right)^l, \left(x_4^*\right)^C, \left(x_4^*\right)^u \right\rangle = \langle 6.37, 6.37, 6.37 \rangle,
\end{cases}
$$

Now, the optimal value of objective function can be obtained. Therefore, the optimal value of the problem may be written as follows:

$$
\left(\tilde{C}^T \tilde{X}^*\right)_{represented\ \ method} = \left\langle \left(C^T x^*\right)^l, \left(C^T x^*\right)^C, \left(C^T x^*\right)^u \right\rangle =
$$
$$
\left\langle \sum_{j=1}^4 \left(C_j x_j^*\right)^l, \sum_{j=1}^4 \left(C_j x_j^*\right)^C, \sum_{j=1}^4 \left(C_j x_j^*\right)^u \right\rangle = \langle 304.6, 509.8, 704.43 \rangle.
$$

Now, using Ezzati's method the optimal solution and optimal value of objective function are given as follows:

$$
\tilde{X}^* = \begin{cases}
\tilde{x}_1^* = \left\langle \left(x_1^*\right)^l, \left(x_1^*\right)^C, \left(x_1^*\right)^u \right\rangle = \langle 17.27, 17.27, 17.27 \rangle, \\
\tilde{x}_2^* = \left\langle \left(x_2^*\right)^l, \left(x_2^*\right)^C, \left(x_2^*\right)^u \right\rangle = \langle 2.16, 2.16, 2.16 \rangle, \\
\tilde{x}_3^* = \left\langle \left(x_3^*\right)^l, \left(x_3^*\right)^C, \left(x_3^*\right)^u \right\rangle = \langle 4.64, 9.97, 16.36 \rangle, \\
\tilde{x}_4^* = \left\langle \left(x_4^*\right)^l, \left(x_4^*\right)^C, \left(x_4^*\right)^u \right\rangle = \langle 6.36, 6.36, 6.36 \rangle,
\end{cases}
$$

and

$$\left(\tilde{C}^T\tilde{X}^*\right)_{Ezzati's\ method}=\left\langle\left(C^Tx^*\right)^l,\left(C^Tx^*\right)^C,\left(C^Tx^*\right)^u\right\rangle=$$
$$\left\langle\sum_{j=1}^4\left(C_jx_j^*\right)^l,\sum_{j=1}^4\left(C_jx_j^*\right)^C,\sum_{j=1}^4\left(C_jx_j^*\right)^u\right\rangle=\langle 304.58,509.79,704.37\rangle,$$

By comparing the results of the proposed method in this note with Ezzati's method, we can conclude that our result is more reliable, since:

$$\langle 304.58,509.79,704.37\rangle=\left(\tilde{C}^T\tilde{X}^*\right)_{Ezzati's\ method}=$$
$$\left\langle\left(C^Tx^*\right)^l,\left(C^Tx^*\right)^C,\left(C^Tx^*\right)^u\right\rangle<\left(\tilde{C}^T\tilde{X}^*\right)_{represented\ \ method}=\left\langle\left(C^Tx^*\right)^l,\left(C^Tx^*\right)^C,\left(C^Tx^*\right)^u\right\rangle=$$
$$\langle 304.65,509.89,704.43\rangle.$$

4. Conclusion

This work concentrated on Definition 2.6 which was proposed by Ezzati et al. (2015) to compare any arbitrary triangular fuzzy numbers. We demonstrated that their presented method is not well in general, thus the suggested approach to find the fuzzy optimal solution of fully fuzzy linear programming problems by Ezzati et al. (2013), can be improved using some general definitions and a new version is presented in this note. The new formulation is validated using an example.

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