# Optimization of Bi-objective Un-equal Area Facility Layout Problem with Grid System Approach by Simulated Annealing 

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#### Abstract

Facility Layout Planning (FLP) is one of the classic and challenging problems in the literature of Operations Research. In this paper, a bi-objective, un-equal area, open-field FLP is investigated. Also, the grid system FLP is considered since there are some complex problems in the real world which need to be modeled discretely. First of all, an integer non-linear mathematical model is proposed to address the grid system and un-equality of the size of the facilities. After that, in order to solve the model, a bi-objective simulated Annealing optimization algorithm is proposed After that, by solving numerical examples, the validation and efficiency of the proposed SA are shown by comparing with exact methods of small- and large-sized problems. In addition, the non-dominated solutions (Pareto optimal set) have been obtained, so a user can choose the desired layout based on his/her opinion. Finally, conclusions and suggestions are proposed.


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Facility layout problem; Un-equal area; Grid system; Simulated Annealing; Pareto optimal set.

## 1. Introduction

Facility Layout Problem (FLP) is one of the classic and challenging problems in Operations Research (OR) field. According to FLP, facilities are objects which we want to locate at proper locations considering well-defined objective functions. In other words, FLP intends to reach the best allocation of facilities to locations in order to achieve productivity and efficiency alongside the constraints of the problem Meller \& Bozer (1996). It had been proven that FLP is an NP-Hard problem Tate* \& Smith (1995), so it must be solved by heuristic or meta-heuristic algorithms in large-size examples. Also, based on different studies, FLP is important in both theory and application.

In fact, FLP will be applicable to many real-world problems if each facility is considered as a machine, a working center, a manufacturing unit, a department, a warehouse, and a sitelevel facility Heragu (2008). These considerations of facilities also will lead to a new applicable problem, for example, by considering the facilities as site-level ones, we will encounter Construction Site Layout Problem (CSLP), which has been investigated in the Civil engineering discipline by many researchers Sadeghpour \& Andayesh (2015).

First of all, we have to understand the existing concepts and definitions of FLP. FLP with the un-equal area facilities is named un-equal area facility layout problem (UA-FLP) in which facilities are free in their size or area Tate $* \&$ Smith (1995). UA-FLP is divided into discrete UA-FLP or a continuous one. By discrete modeling, which is shown in Figure 1, the site floor (where facilities are supposed to be located) is modeled by discrete points or grid systems. On the other hand, in continues one, which is shown in Figure 2, there are no grid cells for modeling the site Hosseini-Nasab, Fereidouni, Ghomi, \& Fakhrzad (2018). In grid system UA-FLP, the shape of each facility can be considered regular (for example rectangular) and irregular (for example Figure 3). In addition, if FLP is mentioned as a dynamic problem, in which layout is for different periods, is named Dynamic Facility Layout Problem (DFLP), vice versa, in the static problem, the layout planning is just in one period Zhu, Balakrishnan, \& Cheng (2017). FLP with the multi-floor layout is called MFLP; on the other hand, FLP is single floor Ahmadi, Pishvaee, \& Jokar (2017). Furthermore, facilities are located according to the material-handling path including single-row, multi-row, double-row, parallel-row, loop, open-field, multi-floor - all of these layouts have been explained clearly by Hosseini-Nasab et al. (2018).


Fig. 1. Discrete representation of layout space


Fig. 2. Continues representation of layout space


Fig. 3. Irregular facilities at the discrete layout space
By and large, in this paper, the grid system FLP is considered since there are some complex problems in the real world which need to be modeled discretely. Moreover, we
consider FLP as multi-objective, un-equal area, rectangular-shaped, single floor, open-field and static. By reviewing the literature in the following, we could see our problem scope among the other FLP classes and what contribution we made.

In the literature, FLP has been studied by many researchers for many decades. These review papers Drira, Pierreval, \& Hajri-Gabouj (2007), Barsegar (2011), Hosseini-Nasab et al. (2018) are sufficient and advised to understand FLP. All the articles in the above review papers with the newest references considered multi-objective, un-equal area, discrete or continues, rectangular-shaped, single floor, open-field, and static are shown in Table 1.

Table 1. Literature review with the classification proposed by Hosseini-Nasab et al. (2018)

| Ref. | Objective function | Rectangular and fixed | Single floor | Material handling system | Static | Layout presentation | Optimization method |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Matai (2015) | Multiobjective | YES | YES | Multirow | YES | Discrete | SA |
| Matai, Singh, \& Mittal (2013) | Multiobjective | YES | YES | Multi- <br> row | YES | Discrete | SA |
| Şahin (2011) | Multiobjective | YES | YES | Multi- <br> row | YES | Discrete | SA |
| Singh \& Singh (2011) | Multiobjective | YES | YES | Multi- <br> row | YES | Discrete | AHP-based heuristic |
| Singh \& Singh (2010) | Multiobjective | YES | YES | Multi- <br> row | YES | Discrete | New heuristic |
| Şahin \& Türkbey (2009) | Multiobjective | YES | YES | Multirow | YES | Discrete | SA-Pareto set |
| $\begin{gathered} \text { Chen * \& Sha } \\ (2005) \end{gathered}$ | Multiobjective | YES | YES | Multi- <br> row | YES | Discrete | New heuristic |
| Suresh \& Sahu (1993) | Multiobjective | YES | YES | Multi- <br> row | YES | Discrete | SA |
| Armour \& Buffa (1963) | Multiobjective | YES | YES | Multi- <br> row | YES | Discrete | New heuristic |
| Lee \& Lee (2002) | Multiobjective | YES | YES | Multi- <br> row | YES | Continuous | Hybrid GA |
| Heragu \& Kusiak (1991) | Multiobjective | YES | YES | Multi- <br> row | YES | Continuous | New heuristic |
| Liu \& Meller (2007) | Multiobjective | YES | YES | Multirow | YES | Continuous | GA |
| MoatariKazerouni, Chinniah, \& Agard (2015) | Multiobjective | YES | YES | Openfield | YES | Discrete | Approximated |
| Kelachankuttu, Batta, \& Nagi (2007) | Singleobjective | YES | YES | Openfield | YES | Discrete | Exact |
| Asl \& Wong (2017) | Singleobjective | YES | YES | Openfield | YES | Continuous | PSO |
| García- |  |  |  |  |  |  |  |
| Hernández, Salas-Morera, GarciaHernandez, | Singleobjective | NO | YES | Multirow | YES | Continuous | Coral Reefs Optimization |
| ```Salcedo-Sanz, & de Oliveira (2019)``` |  |  |  |  |  |  |  |
| Hasda, <br> Bhattacharjya, Bennis, \& Manufacturing | Singleobjective | YES | YES | Openfield | YES | Continuous | Modified GA |


| Cravo, Amaral, |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \& Research <br> (2019) | Single- <br> objective | YES | YES | Single- <br> row | YES | Continuous | Greedy <br> Randomized <br> Adaptive <br> Search <br> Procedure |
| Palomo-Romero, |  |  |  |  |  |  |  |
| SRASP) |  |  |  |  |  |  |  |
|  <br> García- <br> Hernández <br> (2017) | Single- <br> objective | NO | YES | Multi- <br> row | YES | Continuous | Island Model <br> Genetic <br> Algorithm <br> (IMGA) |
| Recent Study | Multi- <br> objective | YES | YES | Open- <br> field | YES | Discrete | SA |

As it is obvious from Table 1, there are five articles which are modeled discretely and solved by SA, while they are in the class of multi-row facility layout problems. Also, Moatari-Kazerouni et al. (2015) is similar to our paper although it considers the occupational health and safety (OHS) factors as objective functions. Furthermore, in these articles, there is no distinct mathematical model for the formulation of the grid system UA-FLP. However, in our paper, we have presented a different mathematical model that is not quadratic assignment problem (QAP) formulation. In other words, we propose the mathematical model that is not only able to represent the grid system space with the un-equal area but is not also formulated by QAP approach, unlike the existing papers. After defining the problem and the model, an efficient multi-objective SA optimization algorithm is also proposed. Totally, the contributions of the current paper can be listed below:

- A new mathematical model for grid system UA-FLP
- Proposing an optimization algorithm based on multi-objective simulated annealing

The main purpose of this paper is to model the grid system UA-FLP and propose an optimization algorithm in order to solve it. In the following, in the 2 nd section, the mathematical model is proposed. Then in the 3rd section, the proposed simulated annealing algorithm is explained. In the 4th section, numerical examples are solved by the proposed method and the results are shown. Finally, in the 5th section, conclusion and future studies are mentioned.

## 2. Problem Formulation

Grid system UA-FLP is shown in Figure 4, in which facilities are size-fixed and rectangular, and also the distance between two facilities is measured by the Euclidean distance between reference points (red points). By these reference points, the representation of the solution space can be presented by Figure 5. Also, it is assumed that there is no permission to the rotation of the facilities, and also the flow of material between facilities has remained constant during the layout process (static layout). Moreover, we have assumed that our material-handling system (MHS) is open-field, in which the facilities can be located everywhere at the site without any constraints imposed by other material-handling paths.


Fig. 4. Grid system UA-FLP with rectangular facilities and how to calculate the distances (vertical and horizontal numbers indicate the reference points)


Fig. 5. The representation of the solution space by the reference points for the grid system of Figure 4
In this section, by Paes, Pessoa, \& Vidal (2017), Hasda, Bhattacharya, \& Bennis (2016) and Şahin \& Türkbey (2009), a new mathematical model of bi-objective grid system UA-FLP has been proposed. In this model, there are two objective functions including material handling cost (MHC) and closeness rating score (CRS), which must be minimized. For our problem, MHC objective function is selected because it is an important criterion in industryrelated problems Tompkins, White, Bozer, \& Tanchoco (2010). Also along with MHC, we have considered CRS as a second criterion as by more objective functions, a decision-maker (DM) will be able to select the desired solution despite conflicting objectives Şahin \& Türkbey (2009). The decision variable ( $u_{i}, v_{i}$ ) is the address of the reference point of the facility $i$, which was shown in Figure 4. Both $u_{i}$ and $v_{i}$ are integer decision variables. Parameter $C_{i j}$ is the cost of material flow per distance unit between facility $i$ and facility $j$. Parameter $f_{i j}$ is the amount of the flow of material between facility $i$ and facility $j$. Parameter $r_{i j}$ is the closeness rate between facility $i$ and facility $j$. Parameter $d_{i j}$ is the Euclidean distance between the reference point of the facility $i$ and the reference point of the facility $j$. Parameter $A_{i j}$ is the common area between facility $i$ and facility $j$. Parameter ( $X_{U}, Y_{U}$ ) is the upper limit of length and width of the site floor. Parameter $\left(X_{L}, Y_{L}\right)$ is the lower limit of length and width of the site floor. Parameter $\left(l_{i}, b_{i}\right)$ is the length and width of the facility $i$. Therefore, the mathematical model can be presented as below:
$\operatorname{Min} \sum_{i=1}^{n} \sum_{j=i+1}^{n} C_{i j} f_{i j} d_{i j}$
$\operatorname{Min} \sum_{i=1}^{n} \sum_{j=i+1}^{n} r_{i j} d_{i j}$
s.t
$\sum_{i=1}^{n} \sum_{j=1}^{n} A_{i j}=0$

$$
\begin{equation*}
i \neq j \tag{3}
\end{equation*}
$$

$u_{i}+l_{i} \leq X_{U}$
$i=1, \ldots, n$
$u_{i} \geq X_{L}$
$i=1, \ldots, n$
$v_{i} \leq Y_{U}-1$
$i=1, \ldots, n$
$v_{i}-b_{i} \geq Y_{L}-1$
$i=1, \ldots, n$

$$
\begin{align*}
& \begin{array}{l}
A_{i j}=\max \left[0,1+\min \left(u_{i}+l_{i}-1, u_{j}+l_{j}-1\right)-\max \left(u_{i}, u_{j}\right)\right] \\
\\
\quad * \max \left[0,1+\min \left(v_{i}, v_{j}\right)-\max \left(v_{i}-b_{i}+1, v_{j}-b_{j}\right.\right. \\
+1)]
\end{array} \\
& \begin{array}{l}
d_{i j}=\sqrt{\left(u_{i}-u_{j}\right)^{2}+\left(v_{i}-v_{j}\right)^{2}} \\
u_{i}, v_{i} \text { integer }
\end{array} \quad i \neq j  \tag{8}\\
&
\end{align*}
$$

The above mathematical model is integer non-linear programming (INLP) that must be solved to attain the optimal location of facilities in the grid system layout. Eq. (1) is the first objective function in which the total cost of material handling between facilities is minimized and Eq. (2) is the second objective function in which the closeness rating multiple distances between facilities is minimized too. Eq. (3) is the constraint that controls the overlapping among the facilities which has to be zero. Eq. (4) to Eq. (7) are the constraints which control the location of facilities in the site floor. Eq. (8) calculates the common area between two facilities which must be zero. Eq. (9) denotes the distance between facilities. Eq. (10) denotes the type of decision variables of the model. It is noteworthy to say that this model represents the UA-FLP as a discrete (grid system) approach which can be solved by MINLP solvers too.

## 3. Methodology

### 3.1. Proposed bi-objective Simulated Annealing (SA) Optimization Algorithm

SA is one of the useful and popular meta-heuristics as it is efficient to solve the combinatorial optimization problems Abbasi, Shadrokh, \& Arkat (2006). Furthermore, SA reaches the nearglobal solution of large combinatorial optimization problems because of its capability of "avoiding getting trapped in local optimum" Suresh \& Sahu (1993). For the first time, SA introduced by Kirkpatrick, Gelatt, \& Vecchi (1983) as a "stochastic search method" and had been inspired by the cooling process of solids. Van Laarhoven \& Aarts (1987) and Ingber (1993) are sufficient to perceive the concept of SA. Also, Mavridou \& Pardalos (1997) is enough to review the implementation of SA in FLP. Moreover, Czyzżak \& Jaszkiewicz (1998) is recommended to researchers who are interested in Pareto SA optimization technique.

To brief the introduction of SA, it begins with the initial solution randomly or greedy. SA has two loops in its process. In the first loop, stopping criteria to decrease the temperature is determined, and then the second loop, which is inside the first loop, determines the total iterations of the algorithm in each temperature. Also, SA does not always reject worse solutions; in fact, it accepts a worse solution with a specific probability by which it avoids trapping in the local optima. The parameters of SA and stopping criteria in this paper for each loop are shown in Table 2. The efficiency of SA depends on the tuning of its parameters, Park \& Kim (1998) has talked about guidelines for the setting parameters briefly.

Table 2. Parameters of SA and stopping criteria in this paper

| Parameter | Notation |
| :---: | :---: |
| Initial temperature | $T_{\max }$ |
| Final temperature | $T_{\min }$ |
| Cooling rate | $\alpha$ |
| Number of iterations in each temperature | $N_{e x}$ |
| The procedure to decrease the temperature in each step | $\dot{T}=\alpha \times T$ |
| Stopping criteria for interior loop (iterations in each |  |
| temperature) | Reaching a specific number of iterations |

In the following, the pseudo-code of the proposed algorithm to solve multi-row UA-FLP with a grid system by bi-objective SA is given. The main code had been written using C++ programming.

1. Input: algorithm starts
$s=s_{0}$; // the feasible solution space is the two-dimensional matrix ( $2 \times$ number of facilities) (each column is the address of the reference points from the first facility to last one (left to right))
$T=T_{\max }$; // final temperature

## 2. Repeat

$k=0 ;$

## Repeat

Generate neighboring solution by changing one of the reference points randomly as below (the feasibility of the generated solution must be checked):

| 1 | 6 | 3 | 9 | 4 | 3 | 7 | 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 5 | 9 | 1 | 2 | 7 | 8 | 4 |  |
| $\sqrt{5}$ |  |  |  |  |  |  |  |  |
| 1 | 6 | 2 | 9 | 4 | 3 | 7 | 1 |  |
| 3 | 5 | 4 | 1 | 2 | 7 | 8 | 4 |  |

$$
\Delta E=f\left(s^{\prime}\right)-f(s)
$$

If $\Delta E \leq 0$ then $\boldsymbol{s}=\boldsymbol{s}$;
Else accept $s$ with the probability $e^{-\Delta E / T}$
$k=k+1$;
Until ( $k \leq N_{e x}$ )
$T=\alpha \times T$; // temperature update
Until ( $T>T_{\text {min }}$ )
3. Output: Display the best solution

Moreover, the multi-objective SA needs to determine the weights of each objective function in order to specify the value of improvement of the solutions in each step. To address this issue, we used Eq. (11) in which $\lambda$ is determined by the user's opinion, if each objective function is more important than others, it will get more weight based on the value of $\lambda$.
$f(s)=\lambda \times \sum_{i=1}^{n} \sum_{j=i+1}^{n} C_{i j} f_{i j} d_{i j}+(1-\lambda) \times \sum_{i=1}^{n} \sum_{j=i+1}^{n} r_{i j} d_{i j}$
Also, it is important to say that the generation of the neighboring solutions is performed by changing the location of the only one facility in each iteration. To summarize, in this section, the proposed SA as an optimization method of open-field UA-FLP with a grid system was discussed. In the following, the concept of Pareto optimal set is defined mathematically.

### 3.2. Pareto Optimal Set

If there are multiple objective functions in a problem, these objective functions show conflicting behavior toward each other. In this case, instead of one optimal solution, there will be an optimal solution set (mathematical model (12)). If the solutions in the optimal set have the properties given in Eq. (13) and Eq. (14), it will be called the Pareto optimal set (non-dominated). Also, the concept of Pareto optimal set is briefly explained by Deb (2014).

$$
\begin{align*}
& \min F(X)=\left\{f_{1}(X), \ldots, f_{n}(X)\right\}  \tag{12}\\
& \text { s.t } \\
& g(X) \leq 0, h(X)=0 \\
& f_{i}\left(X_{1}\right) \geq f_{i}\left(X_{2}\right) \\
& f_{i}\left(X_{1}\right)>f_{i}\left(X_{2}\right)
\end{align*}
$$

## 4. Results and Discussion

In this section, we are going to solve two numerical examples in which the first one is singleobjective (only the MHC), open-field, un-equal area, single-floor, and static problem. This problem has been solved to compare its results with the exact solvers in order to verify the proposed SA method. Inputs of the first example are given in Table 3. In the first example, we have just 3 facilities which are must be located in the site floor, and also these facilities must not have any overlap with each other. The length and width of the site floor are 5 and 5 respectively in this example.

Table 3. Cost of material handling between facilities and size of facilities in the first problem

| Facilities | Cost of material handling $\left(\boldsymbol{C}_{\boldsymbol{i} \boldsymbol{j}} \times \boldsymbol{f}_{\boldsymbol{i} \boldsymbol{j}}\right)$ between facilities | Length | width |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ |  |  |  |  |
| $\mathbf{2}$ | 0 | 1 | 2 | 1 | 2 |
| $\mathbf{3}$ | 1 | 0 | 4 | 3 | 3 |

After solving the first problem by the proposed SA, its results have been compared with the exact solvers of the GAMS software. The results are given in Table 4, which prove the verification of the proposed algorithm and its exactness to solve the small-size problems. From Table 4, it is clear that there are 2 optimal solutions for this problem which have the same value of the objective function.

Table 4. The results obtained by the proposed SA and the other solvers of GAMS software

| Solver | Optimal layout of the facilities |  |  | Objective function | Execution time |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 |  |  |
| SCIP (Full) | 1 | 2 | 0 | 12.2426 | 3 sec . |
|  | 4 | 3 | 3 |  |  |
| LINDOGLOBAL (Demo) | 0 | 2 | 0 | 12.2361* | 0.0001 sec . |
|  | 3 | 2 | 2 |  |  |
| LINDO (Demo) | 0 | 2 | 0 | $12.2361^{*}$ | 0.0001 sec . |
|  | 3 | 2 | 2 |  |  |
| QOUENNE (Full) | - | - | - | No solution returned |  |
| ALPHAECP (Demo) | 0 | 2 | 0 | 14.828 | 2 sec . |
|  | 4 | 2 | 2 |  |  |
| The proposed SA | 0 | 2 | 0 | 12.2361* | 1 sec . |
|  | 4 | 3 | 3 |  |  |

The second example is multi-objective, open-field, un-equal area, single-floor and static with greater size. This problem has been solved to show the efficiency of the proposed SA of the large-size problems. Inputs of the first and second examples are given in Table 5 and 6. In this example, we have 8 facilities that must be located in the specified space and do not have any overlap with each other. The length and width of the site floor are 8 and 8 respectively.

Table 5. Cost of material handling between facilities and size of facilities in the second problem

| Facilities | Cost of material handling $\left(\boldsymbol{C}_{\boldsymbol{i} \boldsymbol{j}} \times \boldsymbol{f}_{\boldsymbol{i} \boldsymbol{j}}\right)$ | between the facilities | length | Width |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ |  |  | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |  |  |
| $\mathbf{1}$ | 0 | 1 | 2 | 0 | 0 | 0 | 2 | 0 | 1 | 2 |
| $\mathbf{2}$ | 1 | 0 | 4 | 3 | 6 | 0 | 0 | 2 | 3 | 3 |
| $\mathbf{3}$ | 2 | 4 | 0 | 2 | 0 | 3 | 1 | 0 | 2 | 2 |
| $\mathbf{4}$ | 0 | 3 | 2 | 0 | 5 | 2 | 0 | 2 | 2 | 2 |
| $\mathbf{5}$ | 0 | 6 | 0 | 5 | 0 | 0 | 0 | 4 | 1 | 2 |
| $\mathbf{6}$ | 0 | 0 | 3 | 2 | 0 | 0 | 4 | 0 | 1 | 1 |
| $\mathbf{7}$ | 2 | 0 | 1 | 0 | 0 | 4 | 0 | 1 | 4 | 4 |
| $\mathbf{8}$ | 0 | 2 | 0 | 2 | 4 | 0 | 1 | 0 | 2 | 3 |

Table 6. Closeness rating score between facilities

| Facilities | Closeness rating score $\left(\boldsymbol{r}_{\boldsymbol{i j}}\right)$ between the facilities |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |  |
| $\mathbf{1}$ | 0 | 7 | 0 | 5 | 4 | 0 | 0 | 1 |  |
| $\mathbf{2}$ | 7 | 0 | 4 | -1 | 2 | 1 | 1 | 2 |  |
| $\mathbf{3}$ | 0 | 4 | 0 | 2 | 2 | -2 | 8 | 4 |  |
| $\mathbf{4}$ | 5 | -1 | 2 | 0 | 1 | 0 | 4 | 1 |  |
| $\mathbf{5}$ | 4 | 2 | 2 | 1 | 0 | 0 | 6 | 4 |  |
| $\mathbf{6}$ | 0 | 1 | -2 | 0 | 0 | 0 | 2 | 8 |  |
| $\mathbf{7}$ | 0 | 1 | 8 | 4 | 6 | 2 | 0 | 9 |  |
| $\mathbf{8}$ | 1 | 2 | 4 | 1 | 4 | 8 | 9 | 0 |  |

To solve of the second example, parameters of SA must be determined. The information on SA parameters is given in Table 7. In fact, by executing different experiments, it has been observed that by increasing the values of $T_{\max }, N_{e x}$ and $\alpha$, the quality of the solutions and computational time had increased. On the other hand, by decreasing the value of $T_{\min }$ the quality and total solution search had increased. Also, these experiments have shown that the execution time is more sensitive to the values of $\alpha$ and $N_{e x}$, which lead to increasing the solution space search. By Table 7, the parameter $\lambda$ is valued as 0.5 because it was assumed that the user had an equal preference towards two objective functions, and the ranges of the values of the SA parameters are given in Table 7 that the experiments were operated on all of the options. The implementation of the proposed algorithm for UA-FLP with a grid system was done by $\mathrm{C}++$ programming using corei 3 and 2.0 GB RAM computer.

Table 7. Best value of SA parameters

| Parameter | $T_{\max }$ | $T_{\min }$ | $\alpha$ | $N_{e x}$ | $\lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Best values based on experiments | 500 | 0.000001 | 0.995 | 50 | 0.5 |
| Range of values | $\{10,500\}$ | $\{0.001,0.000001\}$ | $\{0.9,0.995\}$ | $\{5,50\}$ | 0.5 |

Finally, by solving the second problem, the dominated and non-dominated (Pareto set) solutions were obtained which are shown in Figure 6. So by obtaining the Pareto optimal set, the user will be able to choose the desired layout. In this example, more than 500000 solutions were generated by proposed SA in just 5 seconds and finally, non-dominated solutions (Pareto set) were obtained. It is obvious that by increasing the solution space search, the number of the Pareto set solutions will rise.


Fig. 6. Dominated (blue points) and non-dominated (red curve) solutions obtained by proposed SA - vertical axis: cost, horizontal axis: closeness rate

As shown in Figure 6, a user is able to choose among Pareto set solutions with a red curve that each of these solutions has different cost and closeness rating score. In this example, we suppose that in the Pareto set, the solution which is the nearest to the point $(0,0)$ is the best of all solutions in the user's opinion. So by calculating the distances of nondominated points from point $(0,0)$, the best solution with its values were obtained. Therefore, as given in Figure 7, the optimal facility layout of the sample example was obtained and our proposed Pareto SA algorithm was efficient to solve UA-FLP with the grid system of largesized problem.

| 7 | 7 | 7 | 7 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 7 | 7 | 7 | 7 | 8 | 8 | 8 | 0 |
| 7 | 7 | 7 | 7 | 8 | 8 | 8 | 0 |
| 7 | 7 | 7 | 7 | 5 | 5 | 6 | 0 |
| 3 | 3 | 4 | 4 | 1 | 1 | 0 | 0 |
| 3 | 3 | 4 | 4 | 2 | 2 | 2 | 0 |
| 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 |
| 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 |

Fig. 7. Optimal layout of the second example (large-sized problem)
As a last experiment, we also solve the problems with different sizes in order to test the sensitivity of the proposed optimization algorithm to the increase the problem sizes. The inputs of the problems with different sizes had been provided by random numbers, so in each problem, the values of cost of material handlings and closeness rating scores have been generated randomly from the intervals $[0,9]$ and $[-5,9]$ respectively. Also, the area of each facility has been determined by the random numbers between 1 and 9 . These numbers with random natures would help us reach the results which are able to show us the relationship between the size of the problem and the computational time. Table 8 indicates the impact of the problem size on the execution time of the algorithm. Also, in Table 8, the parameters of SA are the values that were indicated in Table 7. Moreover, in Figure 8, the speediness of the computational time proves the NP-hardness of the grid system UA-FLP by indicating the computational time to increases exponentially.

Table 8. The results of the solution of the problems with different sizes


Fig. 8. The relationship between the size of the problems and the computational time of the algorithm (vertical axis: computational time, horizontal axis: size of the problem)

## 5. Conclusion

In this paper, a new mathematical model of bi-objective grid system (discrete) UA-FLP is proposed. This new model is able to model the grid system and layout of the un-equal size of the facilities. Then this model has been solved by Pareto SA algorithm in which parameters of optimization were set based on their best values. By solving two problems with small and large sizes, the efficiency of the proposed algorithm in comparison with the solvers of GAMS software has been presented, in which the proposed SA was as efficient as other solvers. Also, we can say our proposed multi-objective SA has searched more than 500000 solutions to reach the optimal Pareto set in 5 seconds, which is remarkable due to the complexity of the model. Furthermore, the proposed INLP model of UA-FLP considers the grid cell structure which is important in the problems that have to be modeled discretely. Therefore, the proposed SA showed its capability of solving discrete optimization models.

For future studies, the new proposed model can be solved by other heuristics or metaheuristics such as GA, ACO, PSO, and so on, then solutions which are obtained can be compared with each other. Moreover, to present the problem more real, the irregular shape of the facilities can be considered in the modeling.

## Disclosure statement

No potential conflict of interest was reported by the authors.

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