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Solving two- person zero-sum games with interval-valued fuzzy payoffs

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Abstract

This paper aims to optimize two-person zero-sum matrix games with payoff represented as (γ,δ) interval-valued fuzzy numbers instead of the normal fuzzy numbers. Using the signed distance ranking, the fuzzy payoffs matrix is converted into the corresponding crisp matrix payoffs. Then, a proposed method for solving the problem is presented. Finally, an example is given to illustrate the practical aspect and efficiency of the method.

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1. Introduction

Game theory is a mathematical modeling technique used for decision problems when there are two or more decision makers in conflict or cooperation with each other. Each decision maker plays the game to outsmart the others. Game theory provides many effective and efficient tools and techniques to mathematically formulate and solve many multi person ith strategies in tractions among multiple rational DMs (Krishnaveni and Ganesan, 2018). Game theory is widely applied in many fields, such as economic and management, social policy and international and national policies (Von Neumann and Morgenstern, 1944). Simple necessary and sufficient conditions for the comparison of information structures in zero-sum games have been introduced by Peski (2008). The traditional game theory assumes that all data of game are known exactly by players. However, there are some games in which players are not able to evaluate exactly some data in our realistic situations. In these, the imprecision is due to the inaccuracy of the information and vague comprehension of situations by players. For these, many researchers have made a contribution and introduced some techniques for finding the equilibrium strategies of these games (Berg and Engel, 1998 and Takahashi, 2008).

In many scientific areas, such as system analysis and operations research, a model has to be set up using data which is only approximately known. Fuzzy sets theory, introduced by Zadeh (1965) makes this possible. Dubois and Prade (1980) extended the use of algebraic operations on real numbers to fuzzy numbers using a fuzzification principle. Bellman and Zadeh (1970) introduced the concept of a maximizing decision-making problem. Selvakumari and Lavanya (2015) and Thirucheran et al. (2017) accelerated the fuzzy game.

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Based on the expected value operator and the trust measure of variables under roughness, Xu and Yao (2010) discussed a class of two-person zero-sum games payoffs represented as rough. Campos (1989) studied the game problem under fuzziness in the goal and payoffs. Sakawa and Nishizaki (1994) used max-min principle of game theory to study single and multiobjective matrix games with fuzzy goals and payoffs. Bector et al. (2004) showed that a two-person zero-sum matrix game with having fuzzy goals and fuzzy payoffs are equivalent to a pair of LPPs, each of them is the dual to the other in a fuzzy sense. Vijay et al. (2005) and (2007) based on the fuzzy duality; fuzzy relation approach and ranking function for solving fuzzy matrix games. Pandey and Kumar (2010) proposed a modified approach based on a new order function for solving multi-objective matrix games with vague payoffs. Nan et al. (2010) studied a fuzzy matrix game and a Lexicographic methodology for finding the solution for it. Sahoo (2017) proposed a solution methodology for solving a fuzzy matrix game based on the signed distance method. Li and Hong (2012) proposed an approach for solving constrained matrix games with triangular fuzzy numbers payoffs. Sahoo (2015) introduced a new technique based on the parametric representation of interval numbers to solve the game problem. Bandyopadhyay et al. (2013) studied matrix game with triangular intuitionistic fuzzy number payoff. Bandyopadhyay and Nayak (2013) studied symmetric trapezoidal fuzzy number matrix game payoffs, where they transformed it into different lengths interval fuzzy numbers. Chen and Larboni (2006) defined a matrix game with triangular membership function and proved that two-person zero-sum game with fuzzy payoff matrices is equivalent to two linear programming problems. Seikh et al. (2013) proposed an alternative approach for solving matrix games. Bigdeli et al. (2019) studied fuzzy pay-offs multi-objective security games, where they formulated the problem as fuzzy coefficients bi-level programming.

The remainder of the paper is as follows: Section 2 presents basic concepts and results related to $(\gamma,\ \delta)$ interval-valued fuzzy numbers. Section 3, two-person zero-sum game with $(\gamma,\ \delta)$ interval-valued fuzzy numbers is formulated. Section 4 introduced a proposed method for solving the matrix game. Section 4, a numerical example is given to illustrate the efficiency of the solution approach. Finally, some concluding remarks are reported in sSection 5.

2. Preliminaries

In order to discuss our problem conveniently, basic concepts and results related to fuzzy numbers, and(γ, δ) interval-valued fuzzy numbers are recalled (Chiang, 2001 and Zadeh, 1965).

Definition 1. A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line $\mathbb{R}$ such that:

1. $\mu_{\tilde{A}}(x)$ is piecewise continuous;
2. $\exists\ x \in \mathbb{R}$, with$\mu_{\tilde{A}}(x) = 1$.

Definition 2. If the membership function of the fuzzy set $\tilde{A}$ on $\mathbb{R}$ is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{\gamma\,(x-r)}{(s-r)},\ r < x \le s, \\ \frac{\gamma\,(t-x)}{(t-s)},\ s \le x < t, \\ 0, otherwise, \end{cases}$$

Where $0 < \gamma \le 1$ then $\tilde{A}$ is called a level α fuzzy number and it is denoted as $\tilde{A} = (r, s, t;\ \gamma)$.

Definition 3. An interval- valued fuzzy set $\tilde{A}$ on $\mathbb{R}$ is given by

$\tilde{A} \triangleq \{(x,\ [\mu_{A^-}(x), \mu_{A^+}(x)\ ]): x \in \mathbb{R}\}$, where $\mu_{A^-}(x), \mu_{A^+}(x) \in [0,\ 1]$, and

$\mu_{A^-}(x) \le\ \mu_{A^+}(x); for\ all\ x \in \mathbb{R}$ and is denoted as $\tilde{A} = \left[\tilde{A}^-, \tilde{A}^+\right]$. Let

$$\mu_{\tilde{A}^-}(x) = \begin{cases} \frac{\gamma\,(x-r)}{(s-r)}, & r < x \le s, \\ \frac{\gamma\,(t-x)}{(t-s)}, & s \le x < t, \\ 0, otherwise \end{cases}$$

Then $\tilde{A}^- = (r, s, t; \gamma)$.

Let

$$\mu_{\tilde{A}^+}(x) = \begin{cases} \frac{\delta\,(x-a)}{(s-a)}, & a < x \le s, \\ \frac{\delta\,(b-x)}{(b-s)}, & s \le x < c, \\ 0, otherwise. \end{cases}$$

Then $\tilde{A}^+ = (a, s, b;\ \delta)$.

It is clear that $0 < \gamma \le \delta \le 1$, and $a < r < s < t < b$. Then the interval-valued fuzzy set is defined as

$\tilde{A} \triangleq \{(x, [\mu_{A^-}(x), \mu_{A^+}(x)\ ]) : x \in \mathbb{R}\}$, is denoted as

$\tilde{A} = [(r, s, t; \gamma), (\mathrm{a}, \mathrm{s}, \mathrm{b}; \delta)] = [\tilde{A}^-, \tilde{A}^+]$.

$\tilde{A}$ is called a level (γ, δ) interval- valued fuzzy number

Let, $\tilde{\mathrm{P}} = [(r, s, t; \gamma), (\mathrm{a}, \mathrm{s}, \mathrm{b}; \delta)] \in F_{IVF}(\gamma, \delta)$, and$\tilde{\mathrm{Q}} = [(r_1, s_1, t_1; \gamma), (\mathrm{a}_1, \mathrm{s}_1, \mathrm{b}_1; \delta)] \in F_{IVF}(\gamma, \delta)$. Then the arithmetic operations on $\tilde{\mathrm{P}}$, and $\tilde{\mathrm{Q}}$ are

1. $\tilde{P}(+)\tilde{Q} = [(r + r_1, s + s_1, t + t_1; \gamma), (a + a_1, s + s_1, b + b; \delta)]$,
2. $k\tilde{P} = \begin{cases} [(kr, ks, kt; \gamma), (ka, ks, k\ b; \delta)], & k > 0, \\ [(kt, ks, kr; \gamma), (kb, ks, k\ a; \delta)], & k < 0, \\ [(0,0,0; \gamma), (0, 0,0\ ; \delta)], & k = 0. \end{cases}$

Where, $F_{IVF}(\gamma, \delta) = \{[(r, s, t; \gamma), (a, s, b;\ \delta)] : for\ all\ \ a < r < s < t < b\}$,

$0 < \gamma \le \delta \le 1,$ be the family of (γ, δ) interval- valued fuzzy numbers.

Definition 4. Let $\tilde{\mathrm{P}} = [(r, s, t; \gamma), (\mathrm{a}, \mathrm{s}, \mathrm{b}; \delta)] \in F_{IVF}(\gamma, \delta), 0 < \gamma \le \delta \le 1$. The signed distance of $\tilde{P}$ from $\tilde{0}$ is given as

$$d_0(\tilde{\mathrm{P}}, \tilde{0}) = \frac{1}{8}\left[6\mathrm{s} + \mathrm{r} + \mathrm{t} + 4\mathrm{a} + 4\mathrm{b} + 3(2\mathrm{s} - \mathrm{a} - \mathrm{b})\frac{\gamma}{\delta}\right].$$

Remark 1. $\tilde{\mathrm{P}} = [(a, a, a; \gamma), (\mathrm{a}, \mathrm{a}, \mathrm{a}; \delta)]$, then $d_0(\tilde{\mathrm{P}}, \tilde{0}) = 2a$.

Definition 5. Let $\tilde{\mathrm{P}}, \tilde{\mathrm{Q}} \in F_{IVF}(\gamma, \delta)$, the ranking of level (γ, δ) interval- valued fuzzy numbers in $F_{IVF}(\gamma, \delta)$ using the distance function d_0 is defined as:

$\tilde{\mathrm{Q}} \prec \tilde{\mathrm{P}} \Leftrightarrow \mathrm{d}_0\ (\tilde{\mathrm{Q}}, \tilde{0}\ ) < d_0(\tilde{\mathrm{P}}, 0)$

$\tilde{\mathrm{Q}} \approx \tilde{\mathrm{P}} \Leftrightarrow\ (\tilde{\mathrm{Q}}, \tilde{0}\ ) = d_0(\tilde{\mathrm{P}}, 0)$.

Property 1. Let $\tilde{\mathrm{P}} = [[(r, s, t; \gamma), (\mathrm{a}, \mathrm{s}, \mathrm{b}; \delta)]]$ and $\tilde{\mathrm{Q}} = [(r_1, s_1, t_1; \gamma), (\mathrm{a}_1, \mathrm{s}_1, \mathrm{b}_1; \delta)]$ be (γ, δ) interval- valued fuzzy numbers in$F_{IVF}(\gamma, \delta)$. Then

- $d_0(\tilde{P} \oplus \tilde{Q}, \tilde{0}) = d_0(\tilde{P}, \tilde{0}) + d_0(\tilde{Q}, \tilde{0})$,
- $d_0(k\,\tilde{P}, \tilde{0}) = k\ d_0(\tilde{P}, \tilde{0}), k > 0$.

3. Problem statement and solution concepts

The two-person zero-sum game is the simplest case of game theory in which how much one player receives is equal to how much the other loses. Parthasarathy and Raghavan (2010) studied the case when both players gave pure, mixed strategies. Nevertheless, the noncooperation between players may be vague.

There are three types of two-person zero-sum (γ, δ) interval-valued fuzzy numbers matrix games:

1. Two-person zero-sum matrix games with (γ, δ) interval-valued fuzzy numbers goals,
2. Two-person zero-sum matrix games with (γ, δ) interval-valued fuzzy numbers payoffs,
3. Two- person zero- sum matrix games with (γ, δ) interval-valued fuzzy numbers goals and (γ, δ) interval- valued fuzzy numbers payoffs.

Let us consider a two player zero sum game in which the entries in the payoff matrix $\tilde{A}$ are (γ, δ) interval-valued fuzzy numbers. (γ, δ) Interval-valued fuzzy numbers pay-off matrix is

$$\tilde{A} = \text{Player } I \quad \overset{\text{Player } II}{\begin{pmatrix} \tilde{a}_{11} & \cdots & \tilde{a}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \cdots & \tilde{a}_{mn} \end{pmatrix}} \tag{1}$$

Players I, and II have n and m strategies, respectively denoted by P, and Q, respectively and are defined as

$$P = \{ x \in \mathbb{R}^m : x_i \geq 0, \textstyle\sum_{i=1}^{m} x_i = 1\}, \text{ and} \tag{2}$$

$$Q = \{ y \in \mathbb{R}^n : y_j \geq 0, \textstyle\sum_{j=1}^{n} y_j = 1\}. \tag{3}$$

The mathematical expectation for player I is

$$\tilde{Z} = \textstyle\sum_{j=1}^{n} \sum_{i=1}^{m} \; x_i \tilde{a}_{ij} \, y_j, \tag{4}$$

The mathematical expectation for player II is

$$\tilde{Z} = \textstyle\sum_{i=1}^{m} \sum_{j=1}^{n} \; x_i \, \tilde{a}_{ij} \, y_j. \tag{5}$$

Where, $\tilde{a}_{ij} = \left[\left(a_{ij0}, \; a_{ij1}, a_{ij2}; \; \gamma\right), \left(b_{ij0}, a_{ij1}, b_{ij2}; \; \delta\right)\right]$

Definition 6. (Saddle point): If the min-max value equals to the max-min value then the game is called a saddle point (or equilibrium) and the corresponding strategies are said optimum strategies. The amount of payoff at an equilibrium point is the game value.

Remark 1. It is clear that the two mathematical expectations are the same since the sums are finite.

Because of the vagueness of pay-offs (γ, δ) interval- valued fuzzy numbers, it is very difficult for the players to choose the optimal strategy. So, we consider how to maximize player's or minimize the opponent's fuzzy payoffs. Upon this idea, let us propose the maximum equilibrium strategy as in the following definition.

Definition 7. In one two- person zero- sum game, player I's mixed strategy $x^{\bullet}$ player II's mixed

Strategy $y^{\bullet}$ is considered to be optimal fuzzy strategies if $x^T \tilde{A} \, y^* \leq \; x^{*T} \, \tilde{A} \, y^* \leq x^{*T} \, \tilde{A} \, y$ for any mixed strategies x and y.

Remark 2. The optimal fuzzy strategy of player I is the strategy which maximizes $\tilde{Z}$ irrespective of II's strategy. Also, the optimal fuzzy strategy of player II is the strategy which minimizes $\tilde{Z}$ irrespective of I ' s strategy.

According to definition 5, and based on definition of the score value of each fuzzy payoff $\tilde{a}_{ij}$, the fuzzy payoff matrix defined in (1) is reduced to the classical payoff matrix game as

$$A = \text{Player } I \quad \begin{array}{c} \text{Player} \quad II \\ \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \end{array} \tag{6}$$

Let us consider the game with deterministic payoff matrix (5), and the mixed strategies of players I, and II defined in (2) and (3), respectively. If $F$is the optimum value of the game of a player II , then the linear programming model for player II becomes

$$\min F$$

Subject to

$$\sum_{j=1}^{n} y_j \leq F; y_j \geq 0, j = 1, 2, \dots, n. \tag{7}$$

Putting $y_j' = \frac{y_j}{F}$, then problem (7) becomes

$$\max_{j=1}\left(\sum_{j=1}^{n} y_j'\right)$$

Subject to

$$\sum_{j=1}^{n} a_{ij}\, y_j' \leq 1;\; y_j' \geq 0, j = 1, 2, \dots, n. \tag{8}$$

Similarly, the linear programming model for player I is as

$$\max G$$

Subject to

$$\sum_{i=1}^{m} a_{ij} x_i \geq G; x_i \geq 0, i = 1, 2, \dots, m \tag{9}$$

Putting $x_i' = \frac{x_i}{G}, i = 1,2, \dots, m$. Then problem (9) becomes

$$\min_{i}\left(\sum_{i=1}^{m} x_i'\right)$$

Subject to

$$\sum_{i=1}^{m} a_{ij}\; x_i' \geq 1, x_i' \geq 0, i = 1,2, \dots, m \tag{10}$$

4. Numerical example

Consider the two-person zero-sum matrix game with (γ, δ) interval-valued fuzzy numbers as

$$\tilde{A} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} & \tilde{a}_{14} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} & \tilde{a}_{24} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} & \tilde{a}_{34} \end{pmatrix}$$

Where the values of $\tilde{a}_{ij}$ for $i = 1, 2, 3; j = 1,2,3,4$ are:

$$\tilde{a}_{11} = [(0.5, 1, 5.5; 0.6), (0.25, 1, 7.75; 0.9)], \tilde{a}_{12} = [(13, 14, 15; 0.6), (11, 14, 17; 0.9)],$$
$$\tilde{a}_{13} = [(2, 3, 4; 0.6), (1, 3, 5; 0.9)], \tilde{a}_{14} = [(2, 3, 4; 0.6), (1, 3, 13; 0.9)],$$
$$\tilde{a}_{21} = [(10, 11, 12; 0.6), (9, 11, 13; 0.9)], \tilde{a}_{22} = [(5, 6, 7; 0.6), (3, 6, 9; 0.9)],$$
$$\tilde{a}_{23} = [(3, 5, 7; 0.6), (2, 5, 8; 0.9)], \tilde{a}_{24} = [(10, 11, 12; 0.6), (9, 11, 13; 0.9)],$$
$$\tilde{a}_{31} = [(1, 1, 1; 0.6), (1, 1, 1; 0.9)], \tilde{a}_{32} = [(0.5, 1, 1.5; 0.6), (0.25, 1, 1.75; 0.9)],$$
$$\tilde{a}_{33} = [(17, 20, 21; 0.6), (11, 20, 22; 0.9)], \tilde{a}_{34} = [(1.5, 2,4.5; 0.6), (1, 2, 10; 0.9)]$$

Referring to the signed distance ranking in Definition 4, the above payoff (γ, δ) interval-valued fuzzy numbers matrix game can be reduced to the corresponding deterministic payoff as:

$$A = \begin{pmatrix} 4 & 3 & 6 & 8 \\ 11 & 12 & 10 & 11 \\ 1 & 2 & 19 & 6 \end{pmatrix}$$

According to problem (8), we have

$$\max(y_1' + y_2' + y_3' + y_4')$$

Subject to

$$4y_1' + 3y_2' + 6y_3' + 8y_4' \leq 1,$$

$$11y_1' + 12 + 10y_3' + 11y_4' \leq 1,$$

$$1y_1' + 2y_2' + 19y_3' + 6y_4' \leq 1,$$

$$y_1', y_2', y_3', y_4'.$$

The optimal strategy is:

$$y_1' = 0,\ y_2' = 0,\ y_3' = 0.231,\ y_4' = 0.769$$

In addition, referring to problem (10), we have

$$\min(x_1' + x_2' + x_3')$$

Subject to

$$4x_1' + 11x_2' + 1x_3' \geq 1,$$

$$3x_1' + 12x_2' + 2x_3' \geq 1,$$

$$6x_1' + 10x_2' + 19x_3' \geq 1,$$

$$8x_1' + 11x_2' + 6x_3' \geq 1,$$

$$x_1', x_2', x_3' \geq 0$$

The optimal strategy is:

$x_1' = 0, x_2' = 0.923, x_3' = 0.077.$

Thus the optimal strategies are

$x_1' = 0, x_2' = 0.923, x_3' = 0.077$; $y_1' = 0,\ y_2' = 0,\ y_3' = 0.231,\ y_4' = 0.769$, and the corresponding fuzzy optimal game value is

$[(2.97661, 4.93517, 7.07138; 0.6), (2.24339, 4.93517, 0.79613; 0.9)]$.

5. Concluding Remarks

In this paper, we have considered a two-person zero-sum matrix games with (γ, δ) interval-valued fuzzy numbers. Firstly, we have defined the game with (γ, δ) interval-valued fuzzy numbers pay-offs and then proposed an equilibrium strategy. Secondly, we proposed a solution procedure. Lastly, two numerical examples illustrated our research methods. We have discussed only one kind of game with uncertain payoffs. But of course, there are games with uncertain payoffs which will be take in our consideration the future.

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Conflicts and Interest

The author declares no conflict of interest.

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