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Inventory Model For Retailer-Supplier's Trade Credit Policy

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ABSTRACT

The concept of progressive trade credit optimises financial transactions among the supply chain (SC) partners and provides more control to utilise cash and credit flow. This trade credit effectively manages downstream financing activities such as account receivables and purchase discount schemes. This study attempts to obtain the retailer's robust replenishment policy to optimise the maximum profit. In this work, a progressive trade credit is considered, which suggests that if the vendor settles the financial credit till Mduration, the vendor is not supposed to pay Interest, but if the seller decides the account subsequently M duration however before N ($M<N$), the seller (retailer)has to bear interest amount by the rate Ic_1 . Further, suppose after N duration later account is clear, the seller will bear Interest at the rate Ic_2 ($Ic_2 > Ic_1$). This work tries to develop optimal inventory policy by considering two scenarios: $T < N$ and $T \geq N$. A sensitivity analysis was also conducted to evaluate the strength of the proposed model.

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1. Introduction

Human and societal growth worldwide has increased massive consumption of resources and production activity. Firms keep inventory or stock of the items in the entire supply chain network to satisfy the customers' current and future needs and demands. However, inventory management and control decision-making is critical for organisations in any sector of the economy. Analytical techniques were used to optimise the inventory-related decisions. Ford Harris of Westinghouse Corporation, USA (1913) studied the earliest inventory system model. He developed the classical lot size formula.

Later, scientific thoughts have been utilised in various areas and considered some issues, especially for controlling stock. A standout amongst the most significant concerns of management is choosing when and how much of the amount to arrange or produce so that the all-out expense related to the inventory framework ought to be the least. Furthermore, many researchers have done plenty of work in this area with variability in demand, deterioration and other affected factors. This is reasonably progressively significant when the inventory experiences decay or deterioration. According to a report by the Food and Agriculture Organization (FAO) of the United Nations (FAO), around 40-half of root harvests grown from the ground are squandered yearly, which is the most astounding wastage rate of any food. Likewise, over 40% of losses happened at retail and buyer levels in industrialised nations. These detailed visions uncover the significance of inventory frameworks for decaying things in marketing businesses. Spoilage or decay is categorised by decay or destruction with the ultimate aim of rendering the product unusable for its own reasons. The vast majority of the physical merchandise experiences disintegration after some time. Products, for example, natural products, vegetables, nourishment stuff, and so forth, experience the unkind effects of consumption by direct deterioration while coming up. Electronic products, photography, valuable food, etc. It causes time degeneration due to lack of capacity or continuous electricity use. In this respect, they believe that waste is significant and that scientists should consider it.

Trade credit is a fundamental concept for financing development for numerous organisations. Due to changing business patterns in today's competitive market, the credit period greatly influences acquiring and satisfying new customers. A more extended credit period provides higher working capital to retailers and allows them to enjoy the funds until payback time. Furthermore, within this credit duration, the retailer can collect Interest and gain profit on the trade sales. Conversely, the suppliers provide more credit duration to the retailers to purchase more items. Henceforth, the trade credit delivers a noteworthy job and win-win situation for merchants and retailers in the proposed system. In addition, a progressive credit period benefits both actors, i.e. supplier and retailer. The seller will get more profit from the store; the seller will enjoy this opportunity and get more profit. Hence, this two-stage credit policy positively impacts the customer purchasing more and the supplier as he wants the benefit of more sales. At the same time, his payment will come within the period specified in the credit policy.

Similarly, in highly competitive markets, the diversity of inventory models and their dynamic pricing strategies may support firms in selling their products and generating more revenues to increase demand. Many researchers have formed Traditional inventory models under the idea of time-dependent, price, and stock-dependent demand for many years. Typically, a considerable amount of products in a superstore will lead the client to purchase more merchandise, making the merchandise more prominent. This circumstance persuades the retailer to build their request amount. This phenomenon leads to stock-dependent demand.

Moreover, a good discount on merchandise results in more customers visiting our store. Therefore, rather than having much more stock, it's necessary to balance the selling price while processing the inventory system. So, price-dependent demand is also one of the factors of the demand. It is observed that retailers are quite concerned about trade credit duration since it affects the working capital of the retailers, stock and funds, as well as customer demand. In addition, the effect of loan maturity on consumer demand is discussed, at least in the literature. Still, the loan length is related to the customer's needs. The consequences of the lending period of the request can be immediate or later.

The company's policy is also overstated by inflation and the time value of money. Inflation, an essential macro factor, affects consumer demand and, thus, the stock policy of the company. Higher inflation leads to more spending and less funds available with supply chain partners. Therefore, ignoring inflation or time value of money consideration may cause severe financial risk to supply chain partners, affecting inventory-related decisions. Many researchers have emphasised the importance of inflation on the inventory system in today's global scenario (Buzacott, 1975; Sharma et al., 2019). Hence, considering the inflation factor in inventory modelling provides a safeguard to avoid financial risk and showcase realistic scenarios for inventory-related decisions.

This study framed an EOQ policy for a single decaying product with a progressive credit rule in which the provider offers the retailer a trade credit period to settle the amount, and the retailer offers a credit period to their consumers to accumulate the revenue. The remainder of the proposed paper is composed of pursues. Section 1 highlights the introduction of the study. Related literature has described a few of the finest EOQ policies with different contemporary conditions in section 2. The rest of the paper is sorted out as pursues. Segment 2 offers an audit of the literature to a few of the best inventory models of decaying products and credit strategies under multiple stages. The assumptions and notations utilised throughout the paper are built up in Section 3. Section 4 consists of all our benefit capacities, including the provider (supplier), the retailer and the total profit function for different cases based on altered periods—solution methodology to describe the hypothetical outcomes concerning the benefit capacities created in Segment 5. An illustrative insight with numerical and sensitivity examples is captured in Sections 6 and 7 to validate the results. The final remarks and unique contributions are introduced in Section 8.

2. Literature Review

Deterioration, which alludes to rot, decay, dryness or the vanishing of things, can happen with numerous sorts of products, for example, dairy items, vegetables, medications and natural products (e.g. fruit). A framework of inventory has assumed deterioration as a fundamental issue that occurs frequently over the course of time because items lose their utility after a constant time—the value the executives in time should guzzle to flourish the business world. Consumer loyalty is another valuable part of the obligation endowed to the business houses and firms. Ghare and Schrader (1963) first discussed the concept of deteriorating items based model and demand considered as constant. Onwards, Dave and Patel (1981) demonstrated time-varying demand of a linear nature and constant deterioration. The author also discussed non-deteriorating items, while the rate of deterioration was set at zero with a finite planning horizon. Sana et al. (2004) identified EPQ with a static time horizon and constant deterioration with shortages. Jaggi and Kausar (2009) demonstrated the EOQ inventory model with deteriorating items and a constant deteriorating rate. Annadurai (2013) identified a combined inventory model for constantly deteriorated stuff, considering shortages and credit periods linked with the ordered quantity proposed by the supplier while the demand taken as price-dependent. Lin et al. (2019) described a production-inventory policy with in-transit, i.e. two-stage deterioration and demand at a constant rate. They identified the impact of two-levelled deterioration and obtained the optimum ordered quantity for the supplier and the optimum refill period in case of retailer under the trade credit by proper mechanism.

To attract more customers, it has been seen that a pattern of displayed stock in large quantities creates extreme demand in the superstores. So, inventory models with stock-dependent demand have more importance in current circumstances. Levin et al. (1972) noticed that enormous heaps of customer products shown in a huge store will lead purchasers to buy more. However, such a large number of merchandise heaped up in everybody's manner negatively impacts purchasers and staff similarly. Silver and Peterson (1985) identified that sales volume at a retail store may be directly related to the measure of stock bared on shelves. Teng and Chang (2005) examined the multivariate demand (price and stock-based) in their proposed inventory model. Pal et al.(2006)contemplate the issue of obtaining the heaping volume of demand rate obsessed with displayed stock level and retail price along with commercial charges used to promote the item in the market. Jaggi et al. (2008) developed the linked demand model under two-level trade credit. Chang et al. (2010) examined the inventory model with stock-dependent and selling of the item for consistently decayed items. Roy and Chaudhuri (2012)allowed production for continually deteriorating items by selling price and displayed inventory assumed production per unit for their proposed EPQ model. Onal et al. (2016) developed an inventory system for various perishable products. The demand rate is the function of the selling price and the displayed stock with a fixed shelf life.

Additionally, while credit-induced demands are considered with previous demand policies, it would be more profitable. Banu and Mondal (2016)determined maximum profit with credit's associated demand rate for deteriorating items and supposed ordering cost dependent on the replenishment cycle under an inflationary environment. Recently, Heydari et al. (2017)inventory model was introduced with a credit-dependent demand and synchronised the model in three circumstances. Firstly, in two phases, the retailer offers the credit period to the consumers, and in the last phase, i.e., the third phase, the credit period is allowed to the consumer by the retailer. Meanwhile, the retailer also acquires some period for credit from the supplier; however, the retailer will admit to the credit policy only when their benefit is high.

The vital problem is maintaining the equilibrium among the main actors, i.e., suppliers, producers, sellers, and customers of the supply chain in our globalised world. Kumar and Kumar (2012) introduced an inventory model where the supplier provides two-level trade credit strategies with price-dependent demands. Singh et al. (2013) explained a preservation technology-based inventory model with credit-linked orders, and retailers adapt trade credit financing to survive in the affiliate market and increase sales. The author included selling price in the demand function in their model because cost segregation with a market division approach is progressively used as an increasingly productive valuing strategy. Wu et al. (2014) proposed a model where the supplier provided a two-levelled credit period for fixed-life deteriorated items when the retailer enjoyed the credit period offered by the supplier, and the retailer provided trade credit to the buyer to accumulate profits by increasing the sale. This progressive credit policy positively impacts supply chain actors and is introduced in the EOQ model like Annadurai &Uthayakumar (2015) and Ries et al. (2016). Uthayakumar and Geetha (2018) optimised total inventory cost when the supplier provides a credit period to the retailer with capacity constraints for those items which are not gradually degraded. Kaur (2019) investigated the optimal profitability policy for the retailer to settle the account within the offered credit period. They considered demand probabilistic, resulting in the stock-out situation under default risk.

Yang et al. (2019) incorporated the credit approach to determine the retailer's pricing strategy. In a retailing and industry business, demand can be several types, e.g. stock-dependent, price-based, multivariate etc. (Gupta et al. 2013; Kumar et al. 2017; Sharma et al. 2013; Vashisth et al. 2015; Sharma et al. 2022). Despite this, trade credit policy has become an important part of the present business scenario (Singh et al. 2014; Kumar et al. 2022; Vashisth et al. 2016). Some inventory level demand and variable holding cost (Kumar et al. 2016; Malik et al. 2016) effect on inventory model has an important part of business industry in recent days. Among these strategies, the variable holding cost factors has widely used in business advancement policies (Malik et al. 2016 & 2019; Kumar et al. 2017; Singh and Malik 2010). Tyagi et al. (2023) established an inventory model with Neuro optimisation techniques. The various types of optimisation techniques and procedure explained by Yadav and Malik (2014), and Yadav et al. (2012).

Owing to high cost involved in inventory and stocking of the products and items, one cannot ignore the time value of money. Therefore, inflation would straightforwardly influence the stock arrangement of organisation because of immense venture of capitalinto purchase inventories. This invested capital has high correlation with return on investment (ROI). Therefore, many researchers emphasised to include inflation consideration while making decisions related to inventory of the products and studied the effect of inflation in inventory modelling.Vrat and Padmanabhan (1990) proposed inflation based replenishment model and assumed stock dependent consumption rate. Jolai et al. (2006) built production model for weibull-distribution deteriorated items withinventory level demand. Here they have taken shortages with partial backlogging for finite replenishment under inflation. Singh and Jain (2009) presented an inventory model for supplier's policy to retailer in the context of reverse money under the influence of inflationary environment. Yang et al. (2010) constructed a partially backlogged model and considered time value of money for deteriorating items. Yadav et al. (2015) provided a fuzzy approach to retailer's model under inflation. Consideration of inflation in two storage inventory model for single deteriorating items is studied by Palanivel and Uthayakumar (2016). Ghandehari and Dezhtaherian (2019) attempted a study of optimisation of inventory cost under delayed payment policyfor a single deteriorating item under inflation. They also determined the effect of inflation on basic parameters to handle the marketing strategy and made a comparison with or without shortages.

3. Assumptions and Notations

3.1. Assumptions

The accompanying assumptions and notations are assumed to build up this model:

- Inventory framework manages a single kind of product and replenishment rate is infinite.
- No substitution or fix of deteriorating items during the period viable.
- Demand is depends on the retail price offered by the retailer as well as stock-level.Therefore, the demand function $D(t)$ which is defined as follows $D(t) = a + bI(t) + \alpha p^{-q} + \beta p^{-q}(N - t)$
- Holding cost is $H(t) = h + kt$; $h > 0, k > 0$.
- Delay in payment is allowed under two-stages.
- Time value of money is also modelled.

3.1.1. Notations

A	ordering cost,
$H(t)$	holding cost,
c_d	deteriorating cost per unit item,
$I(t)$	inventory level at any instant of time,
Q	order quantity over at each replenishment cycle when $T < N$,
Q'	order quantity over at each replenishment cycle when $T \geq N$,
c	purchase cost per unit,
p	selling price per unit item where $p > c$,
q	index of elasticity,
θ	rate of deterioration, where $0 < \theta < 1$,
M	first permissible period of delay in settling the accounts with the supplier,
N	second permissible period offered by the supplier, $N > M$,
Ic_1	interest charged per unit timeby the supplier when seller pays during $[M, N]$,
Ic_2	interest charged per unit time by the supplier when seller pays during $[N, T]$,
I_e	interest earned per unit time,
T	cycle time for replenishment,
r	rate of inflation.

4. Mathematical Formulation of the Proposed Model

In this portion, an EOQ inventory model for constant deteriorating items is proposed under two different cases based on second credit period offered by the supplier i.e. **Case I:** $T < N$ and **Case II**: $T \geq N$.

Case I: When $T < N$

At t = 0, Q units enters in the system,the on-hand inventory decreases due to demand $D(t) = \left(a + bI(t) + \alpha p^{-q} + \beta p^{-q}(N - t)\right)$ and the instant state of inventory $I(t)$ at time t is:

$$I'(t)+\theta I(t)=-\left(a+bI(t)+\alpha p^{-q}+\beta p^{-q}(N-t)\right) \qquad 0\le t\le T \tag{1}$$

with the initial and boundary conditions $I(0) = Q$ and $I(T) = 0$ respectively. Consequently, solution of equation (4.1) is

$$I(t)=\left[\frac{a+\alpha p^{-q}}{\theta+b}+\beta p^{-q}\left(\frac{N-t}{\theta+b}+\frac{1}{(\theta+b)^2}\right)\right]\left(e^{(\theta+b)(T-t)}-1\right) \qquad 0\le t\le T \tag{2}$$

Hence the total order quantity per cycle is

$$Q=\left[\frac{a+\alpha p^{-q}}{\theta+b}+\beta p^{-q}\left(\frac{N}{\theta+b}+\frac{1}{(\theta+b)^2}\right)\right]\left(e^{(\theta+b)T}-1\right) \tag{3}$$

Now, the cost components are as follows:

Ordering Cost is

$$OC=A \tag{4}$$

Purchasing Cost is

$$PC=c.Q \tag{5}$$

Holding cost in the total cycle is given in this form

$$HC=\int_0^T (h+kt)I(t)e^{-rt}dt$$

$$HC=he^{(\theta+b)T}\left[\left(\frac{a+\alpha p^{-q}}{(\theta+b)(\theta+b+r)}+\frac{\beta p^{-q}}{(\theta+b)^2(\theta+b+r)}\right)(1-e^{-(\theta+b+r)T})+\frac{\beta p^{-q}}{\theta+b}\right.$$
$$\left.\left(\frac{(N-T)e^{-(\theta+b+r)T}-N}{-(\theta+b+r)}-\frac{1-e^{-(\theta+b+r)T}}{(\theta+b+r)^2}\right)\right]-h\left[\left(\frac{a+\alpha p^{-q}}{(\theta+b)}+\frac{\beta p^{-q}}{(\theta+b)^2}\right)\left(\frac{1-e^{-rT}}{r}\right)\right.$$
$$\left.+\frac{\beta p^{-q}}{\theta+b}\left(\frac{(N-T)e^{-rT}-N}{-r}+\frac{e^{-rT}-1}{r^2}\right)\right]+ke^{(\theta+b)T}\left[\left(\frac{a+\alpha p^{-q}}{(\theta+b)}+\frac{\beta p^{-q}}{(\theta+b)^2}\right)\right.$$
$$\left.\left(\frac{Te^{-(\theta+b+r)T}}{-(\theta+b+r)}-\frac{e^{-(\theta+b+r)T}-1}{(\theta+b+r)^2}\right)+\frac{\beta p^{-q}}{\theta+b}\left(\frac{(N-T)Te^{-(\theta+b+r)T}}{-(\theta+b+r)}-\frac{(N-2T)e^{-(\theta+b+r)T}-N}{(\theta+b+r)^2}+\frac{2e^{-(\theta+b+r)T}-2}{(\theta+b+r)^3}\right)\right]$$
$$-k\left[\left(\frac{a+\alpha p^{-q}}{(\theta+b)}+\frac{\beta p^{-q}}{(\theta+b)^2}\right)\left(-\frac{Te^{-rT}}{r}+\frac{e^{-rT}-1}{r^2}\right)+\frac{\beta p^{-q}}{\theta+b}\left(\frac{(N-T)Te^{-rT}}{-r}+\frac{(N-2T)e^{-rT}-N}{-r^2}+\frac{2e^{-rT}-2}{r^3}\right)\right] \tag{6}$$

Deterioration cost is

$$DC=c_d\left[Q-\int_0^T D(t)dt\right]\int_0^T e^{-rt}dt$$

$$DC = c_d\left[Q - \left(a + \alpha p^{-q} + \beta p^{-q}\left(NT - \frac{T^2}{2}\right)\right) - b\left(\frac{a + \alpha p^{-q}}{(\theta + b)} + \frac{\beta p^{-q}}{(\theta + b)^2}\right)\left(\frac{1 - e^{-(\theta+b)T}}{-(\theta + b)}\right)\right.$$
$$-\frac{b\beta p^{-q}}{\theta + b}\left(\frac{(N - T)e^{-(\theta+b)T} - N}{-(\theta + b)} - \frac{1 - e^{-(\theta+b)T}}{(\theta + b)^2}\right) + \frac{b(a + \alpha p^{-q})T}{(\theta + b)} - \frac{b\beta p^{-q}}{\theta + b}\left(NT - \frac{T^2}{2}\right)$$
$$\left.+\frac{b\beta p^{-q}T}{(\theta + b)^2}\right]\left(\frac{1 - e^{-rT}}{r}\right) \tag{7}$$

Generated Sales Revenue is given by

$$SR = p\int_0^T D(t)e^{-rt}dt$$

$$SR = p\left[(a + \alpha p^{-q})\left(\frac{1 - e^{-rT}}{r}\right) + \left(\beta p^{-q}\left(\frac{(N - T)e^{-rT} - N}{-r} + \frac{e^{-rT} - 1}{r^2}\right)\right)\right.$$
$$+ be^{(\theta+b)T}\left(\frac{(a + \alpha p^{-q})(1 - e^{-(\theta+b+r)T})}{(\theta + b)(\theta + b + r)} + \frac{\beta p^{-q}(1 - e^{-(\theta+b+r)T})}{(\theta + b)^2(\theta + b + r)} + \frac{\beta p^{-q}}{\theta + b}\right.$$
$$\left.\left(\frac{(N - T)e^{-(\theta+b+r)T} - N}{-(\theta + b + r)} + \frac{(e^{-(\theta+b+r)T} - 1)}{(\theta + b + r)^2}\right)\right) - b\left(\left((a + \alpha p^{-q}) + \frac{\beta p^{-q}}{(\theta + b)^2}\right)\left(\frac{1 - e^{-rT}}{r}\right)\right.$$
$$\left.\left.+\frac{\beta p^{-q}}{\theta + b}\left(\frac{(N - T)e^{-rT} - N}{-r} + \frac{e^{-rT} - 1}{r^2}\right)\right)\right] \tag{8}$$

The calculation of Interest and Interest depends on the length of the T period and the M and N loan periods granted, two situations arise:

<u>Sub-case I</u>: When $T < M < N$, Figure 1 represents inventory level versus time.

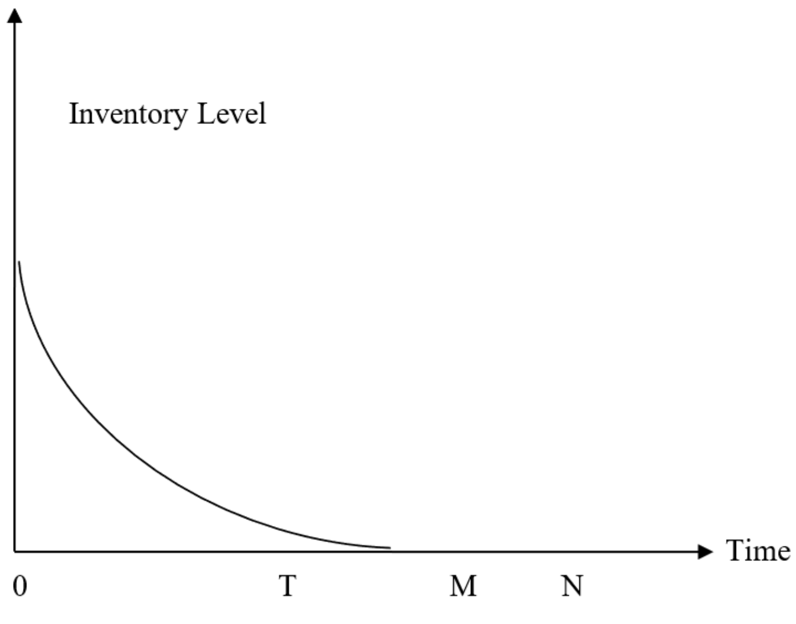

Fig. 1. Inventory level versus time when $T < M < N$.

Here in this case $T < M < N$, the length of the replenishment period is assumed to be shorter than the seller's credit period. The seller therefore does not charge any interest, i.e. interest charges are zero. $IC_1 = 0.$

(9)

However, during the period [0, T], the seller sells Q units and deposits the proceeds in an account that earns I_e. During the period $[T, M]$ the seller keep on depositing the total revenue in to the account to earn Interest.

$$IE_1 = pI_e\left[\int_0^T t.D(t)e^{-rt}\,dt + (M-T)e^{-rT}\int_0^T D(t)\,dt\right]$$

$$= pI_e\left[\left[\left((1+b)(a+\alpha p^{-q}) + \frac{b\beta p^{-q}}{(\theta+b)^2}\right)\left(\frac{Te^{-rT}}{-r} - \frac{e^{-rT}-1}{r^2}\right) + \left(1+\frac{b}{(\theta+b)}\right)\beta p^{-q}\right.\right.$$

$$\left(\left(NT - \frac{T^2}{2}\right)\frac{e^{-rT}}{-r} - \frac{(N-2T)e^{-rT}-N}{r^2} + \frac{2(e^{-rT}-1)}{r^3}\right) + \frac{\beta p^{-q}e^{(\theta+b)T}}{(\theta+b)^2}$$

$$+\frac{(N-2T)e^{-(\theta+b+r)T}-N}{-(\theta+b+r)^2} + \frac{2(e^{-(\theta+b+r)T}-1)}{(\theta+b+r)^3}\Bigg) + (M-T)e^{-rT}\Big\{(a+\alpha p^{-q})T$$

$$+\left(1+\frac{b}{(\theta+b)}\right)\beta p^{-q}\left(NT-\frac{T^2}{2}\right) + \left(b\frac{(a+\alpha p^{-q})}{(\theta+b)} + \frac{b\beta p^{-q}}{(\theta+b)^2}\right)\frac{1-e^{(\theta+b)T}}{-(\theta+b)}$$

$$\left.\left. -\frac{b(a+\alpha p^{-q})T}{(\theta+b)} - \frac{b\beta p^{-q}T}{(\theta+b)^2} + \frac{b\beta p^{-q}}{(\theta+b)}\left(\frac{(N-T)e^{-(\theta+b)T}-N}{-(\theta+b)} + \frac{(e^{-(\theta+b)T}-1)}{(\theta+b)^2}\right)\right\}\right] \quad (10)$$

Total average profit per cycle = [Sales-revenue – Ordering cost – Purchasing cost - Holding cost - Deterioration cost –Interest charged + Interest earned during the cycle].

So, the total inventory profit per unit time is

$$TP_1 = \frac{1}{T}\left[SR - OC - PC - HC - DC - IC_1 + IE_1\right] \quad (11)$$

Sub-case II: $M \le T < N$

In this sub-case $M \le T < N$, the supplier starts to charge the seller for unpaid balance having interest rate Ic_1 at time M (Figure 2).

The Interest payable per unit time is

$$IC_2 = cIc_1\int_M^T I(t)e^{-rt}\,dt$$

$$IC_2 = cIc_1 e^{(\theta+b)T}\left[\left(\frac{a+\alpha p^{-q}}{(\theta+b)(\theta+b+r)}+\frac{\beta p^{-q}}{(\theta+b)^2(\theta+b+r)}\right)(e^{-(\theta+b+r)T}-e^{-(\theta+b+r)M})+\frac{\beta p^{-q}}{\theta+b}\right.$$

$$\left.\left(\frac{(N-T)e^{-(\theta+b+r)T}-(N-M)e^{-(\theta+b+r)M}}{-(\theta+b+r)}+\frac{(e^{-(\theta+b+r)T}-e^{-(\theta+b+r)M})}{(\theta+b+r)^2}\right)\right]-cIc_1\left[\left(\frac{a+\alpha p^{-q}}{(\theta+b)}+\right.\right.$$

$$\left.\left.\frac{\beta p^{-q}}{(\theta+b)^2}\right)\left(\frac{e^{-rT}-e^{-rM}}{-r}\right)+\frac{\beta p^{-q}}{\theta+b}\left(\frac{(N-T)e^{-rT}-(N-M)e^{-rM}}{-r}+\frac{e^{-rT}-e^{-rM}}{r^2}\right)\right] \quad (12)$$

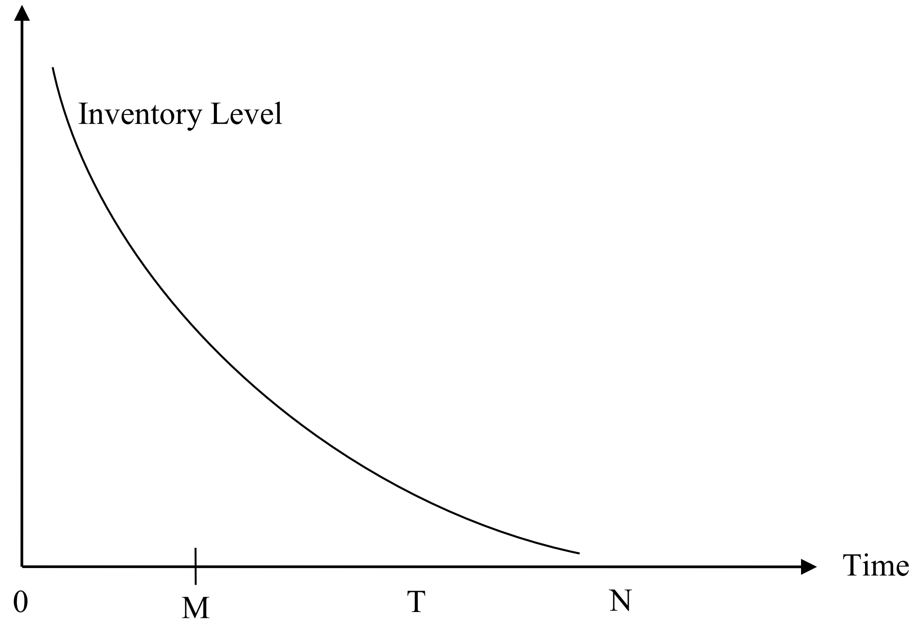

Fig. 2. Inventory level versus time when $M \le T < N$.

Throughout the period [0, M], the seller sells products and credits the revenue in to an account that earns aninterest of I_e. Therefore, the Interest earned in the period [0, M] is

$$IE_2 = pI_e \int_0^M t.D(t)\, e^{-rt}\, dt$$

$$IE_2 = pI_e\left[\left((1+b)(a+\alpha p^{-q})+\frac{b\beta p^{-q}}{(\theta+b)^2}\right)\left(\frac{Me^{-rM}}{-r}-\frac{e^{-rM}-1}{r^2}\right)+\left(1+\frac{b}{(\theta+b)}\right)\beta p^{-q}\right.$$

$$\left(\left(NM-\frac{M^2}{2}\right)\frac{e^{-rM}}{-r}-\frac{(N-2M)e^{-rM}-N}{r^2}+\frac{2(e^{-rM}-1)}{r^3}\right)+\frac{\beta p^{-q}e^{(\theta+b)M}}{(\theta+b)^2}$$

$$\left(\frac{Me^{-(\theta+b+r)M}}{-(\theta+b+r)}+\frac{1-e^{-(\theta+b+r)M}}{(\theta+b+r)^2}\right)+\frac{b\beta p^{-q}e^{(\theta+b)M}}{(\theta+b)}\left(\left(NM-\frac{M^2}{2}\right)\frac{e^{-(\theta+b+r)M}}{-(\theta+b+r)}\right.$$

$$\left.\left.+\frac{(N-2M)e^{-(\theta+b+r)M}-N}{-(\theta+b+r)^2}+\frac{2(e^{-(\theta+b+r)M}-1)}{(\theta+b+r)^3}\right)\right] \quad (13)$$

Total average profit per cycle = [Sales-revenue – Ordering cost – Purchasing cost - Holding cost - Deterioration cost –Interest charged +Interest earned during the cycle].

So, the total inventory profit per unit time is

$$TP_2 = \frac{1}{T}\left[SR - OC - PC - HC - DC - IC_2 + IE_2\right] \tag{14}$$

Case II: When $T \geq N$

Inventory I(t) decreases rapidly during periods of increase due to cash and credit $D(t) = \left(a + bI(t) + \alpha p^{-q} + \beta p^{-q}(N-t)\right)$ up to period N and after that it declines only due to cash demand $\left(a + bI(t) + \alpha p^{-q}\right)$ as well as constant deterioration.Hence,the differential equations of *I(t)* are given by

$$I_1'(t) + \theta I_1(t) = -\left(a + bI_1(t) + \alpha p^{-q} + \beta p^{-q}(N-t)\right); \qquad 0 \leq t \leq N \tag{15}$$

$$I_2'(t) + \theta I_2(t) = -\left(a + bI_2(t) + \alpha p^{-q}\right); \qquad N \leq t \leq T \tag{16}$$

with these boundary conditions $I_1(0) = Q', I_2(T) = 0 \; and \; I_1(N) = I_2(N)$, solutions of Equations (15) and (16) are

$$I_1(t) = -\frac{a + \alpha p^{-q}}{\theta + b} - \beta p^{-q}\left(\frac{N-t}{\theta + b} + \frac{1}{(\theta + b)^2}\right) + e^{-(\theta+b)t}\left(\frac{a + \alpha p^{-q}}{\theta + b}\left(e^{(\theta+b)T} - e^{(\theta+b)N}\right) + \left(\frac{a + \alpha p^{-q}}{\theta + b} + \frac{\beta p^{-q}}{(\theta + b)^2}\right)\left(e^{(\theta+b)N} - 1\right) - \frac{\beta p^{-q} N}{(\theta + b)} + \frac{a + \alpha p^{-q}}{\theta + b} + \beta p^{-q}\left(\frac{N}{\theta + b} + \frac{1}{(\theta + b)^2}\right)\right)$$

(17)

$$I_2(t) = \frac{a + \alpha p^{-q}}{\theta + b}\left(e^{-(\theta+b)(T-t)} - 1\right) \tag{18}$$

Hence the total order quantity per cycle is given in this form

$$Q' = \frac{a + \alpha p^{-q}}{\theta + b}\left(e^{(\theta+b)T} - e^{(\theta+b)N}\right) + \left(\frac{a + \alpha p^{-q}}{\theta + b} + \frac{\beta p^{-q}}{(\theta + b)^2}\right)\left(e^{(\theta+b)N} - 1\right) - \frac{\beta p^{-q} N}{(\theta + b)} \tag{19}$$

Now, the costs associated are

- Ordering Cost

$$OC = A \tag{20}$$

- Purchasing Cost

$$PC = c.Q'. \tag{21}$$

The holding cost in the total cycle is given in this form

$$HC = \int_0^N (h + kt) I_1(t) e^{-rt} dt + \int_N^T (h + kt) I_2(t) e^{-rt} dt$$

$$HC = h\Bigg[\left(\left(\frac{a+\alpha p^{-q}}{(\theta+b)}+\frac{\beta p^{-q}}{(\theta+b)^2}\right)\frac{e^{-rN}-1}{r}-\left(\frac{a+\alpha p^{-q}}{(\theta+b)}+Q^{'}\right)\frac{e^{-(\theta+b+r)N}-1}{(\theta+b+r)}\right.$$

$$+\beta p^{-q}\left(\frac{N}{(\theta+b)}+\frac{1}{(\theta+b)^2}\right)\frac{e^{-(\theta+b+r)N}-1}{(\theta+b+r)}-\frac{\beta p^{-q}}{\theta+b}\left(\frac{N}{r}+\frac{e^{-rN}-1}{r^2}\right)$$

$$\left.+\frac{(a+\alpha p^{-q})e^{(\theta+b)T}}{(\theta+b)}\left(\frac{e^{-(\theta+b+r)T}-e^{-(\theta+b+r)N}}{-(\theta+b+r)}\right)+\frac{(a+\alpha p^{-q})}{(\theta+b)}\frac{e^{-rT}-e^{-rN}}{r}\right]$$

$$+k\left[\frac{a+\alpha p^{-q}}{(\theta+b)}\left(\frac{Ne^{-rN}}{r}+\frac{e^{-rN}-1}{r^2}\right)+\frac{\beta p^{-q}}{(\theta+b)^2}\left(\frac{(Nr+1)(e^{-rN}-1)}{r^2}\right)\right.$$

$$+\left(\frac{a+\alpha p^{-q}}{(\theta+b)}+Q^{'}-\frac{N\beta p^{-q}}{\theta+b}-\frac{\beta p^{-q}}{(\theta+b)^2}\right)\left(\frac{Ne^{-(\theta+b+r)N}}{(\theta+b+r)}+\frac{e^{-(\theta+b+r)N}-1}{(\theta+b+r)^2}\right)$$

$$+\beta p^{-q}\left(\frac{-Ne^{-rN}-N}{(\theta+b)r^2}+\frac{2(1-e^{-rN})}{r^3}\right)+\frac{(a+\alpha p^{-q})e^{(\theta+b)T}}{(\theta+b)}\left(\frac{Te^{-(\theta+b+r)T}-Ne^{-(\theta+b+r)N}}{-(\theta+b+r)}\right.$$

$$\left.\left.-\frac{e^{-(\theta+b+r)T}-e^{-(\theta+b+r)N}}{(\theta+b+r)^2}\right)+\frac{(a+\alpha p^{-q})}{(\theta+b)}\left(\frac{Te^{-rT}-Ne^{-rN}}{r}+\frac{e^{-rT}-e^{-rN}}{r^2}\right)\right]\Bigg] \tag{22}$$

The deterioration cost is given by

$$DC = c_d\left[Q^{'}-\int_0^N D(t)\,dt-\int_N^T D(t)\,dt\right]\int_0^T e^{-rt}dt$$

$$DC = c_d\left[Q^{'}-\left\{N(a+\alpha p^{-q})+\frac{\beta p^{-q}N^2}{2}\left(1-\frac{b}{\theta+b}\right)-\frac{b(a+\alpha p^{-q})N}{(\theta+b)}-\frac{b\beta p^{-q}N}{(\theta+b)^2}\right.\right.$$

$$+b\left(\frac{a+\alpha p^{-q}}{(\theta+b)}+Q^{'}\right)\frac{1-e^{-(\theta+b)N}}{(\theta+b)}+b\beta p^{-q}\left(\frac{N}{\theta+b}+\frac{1}{(\theta+b)^2}\right)\frac{1-e^{-(\theta+b)N}}{(\theta+b)}$$

$$\left.\left.+(T-N)(a+\alpha p^{-q})+b\left(\frac{a+\alpha p^{-q}}{(\theta+b)}\right)\left(\frac{e^{(\theta+b)(T-N)}-1}{(\theta+b)}-(T-N)\right)\right\}\right]\frac{1-e^{-rT}}{r} \tag{23}$$

Generated Sales Revenue is given by

$$SR = p\left[\int_0^N D(t)e^{-rt}dt+\int_N^T D(t)e^{-rt}dt\right]$$

$$SR = p\left[(a+\alpha p^{-q})\left(\frac{1-e^{-rN}}{r}\right)+\beta p^{-q}\left(\frac{N}{r}+\frac{e^{-rN}-1}{r^2}\right)+b\left\{\left(\frac{a+\alpha p^{-q}}{(\theta+b)}+\frac{\beta p^{-q}}{(\theta+b)^2}\right)\right.\right.$$

$$\frac{e^{-rN}-1}{r}-\left(\frac{a+\alpha p^{-q}}{(\theta+b)}+Q^{'}\right)\frac{e^{-(\theta+b+r)N}-1}{(\theta+b+r)}+\beta p^{-q}\left(\frac{N}{(\theta+b)}+\frac{1}{(\theta+b)^2}\right)\frac{e^{-(\theta+b+r)N}-1}{(\theta+b+r)}$$

$$\left.-\frac{\beta p^{-q}}{\theta+b}\left(\frac{N}{r}+\frac{e^{-rN}-1}{r^2}\right)\right\}+(a+\alpha p^{-q})\frac{e^{-rN}-e^{-rT}}{r}+\frac{(a+\alpha p^{-q})be^{(\theta+b)T}}{(\theta+b)}$$

$$\left.\left(\frac{e^{-(\theta+b+r)T}-e^{-(\theta+b+r)N}}{-(\theta+b+r)}\right)+\frac{(a+\alpha p^{-q})b}{(\theta+b)}\frac{e^{-rT}-e^{-rN}}{r}\right] \tag{24}$$

In this case we calculate the Interest earned, and Interest charged in the followingpossible sub-case $M < N \leq T$.

Sub-case: $M < N \leq T$ (Figure 3)

Because there is not enough money in the seller's account to cover the entire purchase price during period M. It pays only a small amount in period M and a small amount in period N. That is, the seller paid interest rate I_{c_1} to the seller $[M, N]$ and the interest rate after N is I_{c_2}.

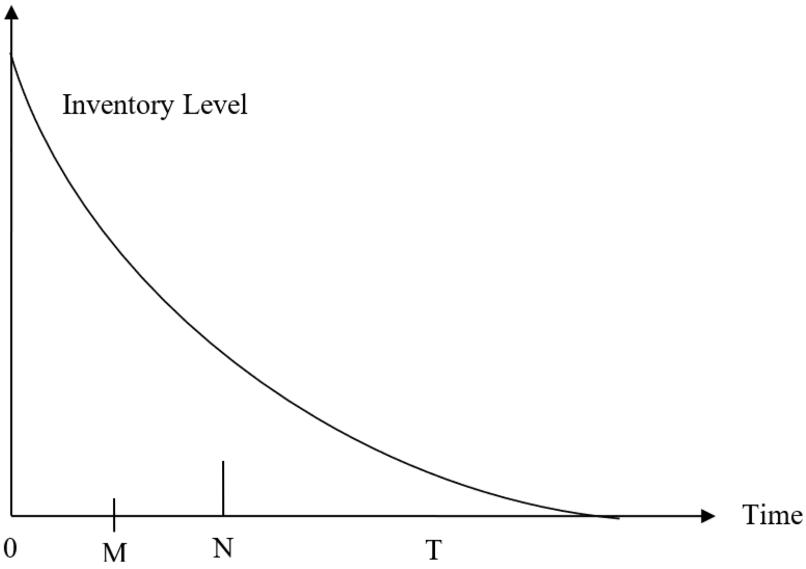

Fig. 3. Inventory level versus time when $M < N \leq T$.

Consequently, the interest payable peryear and the Interest earned is

$$IC_3 = c\left[Ic_1 \int_M^N I_1(t)e^{-rt}\,dt + Ic_2 \int_N^T I_2(t)e^{-rt}\,dt \right]$$

$$\begin{aligned} IC_3 = cIc_1 \Bigg[& \left(\left(\frac{a+\alpha p^{-q}}{(\theta+b)} + \frac{\beta p^{-q}}{(\theta+b)^2} \right) \frac{e^{-rN}-e^{-rM}}{r} - \left(\frac{a+\alpha p^{-q}}{(\theta+b)} + Q' \right) \frac{e^{-(\theta+b+r)N}-e^{-(\theta+b+r)M}}{(\theta+b+r)} \right. \\ & \left. + \beta p^{-q} \left(\frac{N}{(\theta+b)} + \frac{1}{(\theta+b)^2} \right) \frac{e^{-(\theta+b+r)N}-e^{-(\theta+b+r)M}}{(\theta+b+r)} - \frac{\beta p^{-q}}{\theta+b} \left(\frac{(N-M)e^{-rM}}{r} + \frac{e^{-rN}-e^{-rM}}{r^2} \right) \right] \\ & + cIc_2 \left[\frac{(a+\alpha p^{-q})e^{(\theta+b)T}}{(\theta+b)} \frac{\left(e^{-(\theta+b+r)N}-e^{-(\theta+b+r)T}\right)}{(\theta+b+r)} + \frac{a+\alpha p^{-q}}{(\theta+b)} \frac{e^{-rN}-e^{-rM}}{r} \right] \Bigg] \end{aligned} \tag{25}$$

$$IE_3 = pI_e \int_0^M t.D(t)\, e^{-rt}\,dt$$

$$\begin{aligned} IE_3 = pI_e \Bigg[& \left((1+b)(a+\alpha p^{-q}) + \frac{b\beta p^{-q}}{(\theta+b)^2} \right) \left(\frac{Me^{-rM}}{-r} - \frac{e^{-rM}-1}{r^2} \right) + \left(1 + \frac{b}{(\theta+b)} \right) \beta p^{-q} \\ & \left(\left(NM - \frac{M^2}{2} \right) \frac{e^{-rM}}{-r} - \frac{(N-2M)e^{-rM}-N}{r^2} + \frac{2(e^{-rM}-1)}{r^3} \right) + \frac{\beta p^{-q} e^{(\theta+b)M}}{(\theta+b)^2} \end{aligned}$$

$$\left(\frac{Me^{-(\theta+b+r)M}}{-(\theta+b+r)}+\frac{1-e^{-(\theta+b+r)M}}{(\theta+b+r)^2}\right)+\frac{b\beta p^{-q}e^{(\theta+b)M}}{(\theta+b)}\left(\left(NM-\frac{M^2}{2}\right)\frac{e^{-(\theta+b+r)M}}{-(\theta+b+r)}\right.$$

$$\left.\left.+\frac{(N-2M)e^{-(\theta+b+r)M}-N}{-(\theta+b+r)^2}+\frac{2(e^{-(\theta+b+r)M}-1)}{(\theta+b+r)^3}\right)\right] \quad (26)$$

Total average profit per cycle = Sales-revenue – Ordering cost – Purchasing cost - Holding cost - Deterioration cost –Interest charged + Interest earned during the cycle.

So, the total profit per unit time is

$$TP_3=\frac{1}{T}\left[SR-OC-PC-HC-DC-IC_3+IE_3\right] \quad (27)$$

5. Solution Procedure

The aim is to obtain the optimum solution for product planning. The total average revenue per unit (TP) of the system is obtained as follows:

$$TP^*(p,T)=Max.\begin{cases}TP_1(p,T), & T<M<N\\ TP_2(p,T), & M\le T<N\\ TP_3(p,T), & M<N\le T\end{cases} \quad (28)$$

- Here, determine the optimum values of p, T and $TP(p, T)$ in each sub-case with the help of the calculus method by applying necessary and sufficient conditions to maximisation

$$\frac{\partial TP_i(p,T)}{\partial p}=0, \qquad \frac{\partial TP_i(p,T)}{\partial T}=0 \quad (29)$$

$$\frac{\partial^2 TP_i(T,T_1)}{\partial p^2}<0, \frac{\partial^2 TP_i(p,T)}{\partial p^2}\frac{\partial^2 TP_i(p,T)}{\partial T^2}-\left(\frac{\partial^2 TP_i(p,T)}{\partial p\partial T}\right)^2>0 \quad (30)$$

  where i=1, 2 and 3.

- With Mathematica-9.0, find the p* and T* and total profit $TP^*(p, T)$ by equation (29), and for validation of maximisation, equation (30) must be satisfied.
- By illustrating the numerical example for each sub-case. By the help of equation (28), we can maximise the value of profit function $TP^*(p, T)$ for our proposed inventory system.
- '*' denotes the optimal values of respective parameters.

6. Numerical Example

CaseI-<u>Sub-case I</u>: When $T<M<N$

To demonstrate the model the following inventory parameters in appropriate units:

$A=20$, $a=100$, $b=0.4$, $c=1.8$, $q=1.2$, $h=0.2$,
$k=0.2$, $\theta=0.01$, $\alpha=40$, $\beta=50$, $c_d=0.2$, $r=0.5$,
$M=1.5$, $N=3$, $Ic_1=0.18$ $I_e=0.12$.

By Eq. (11), we get optimum values of p^{*}= 8.4020, $T^{*} = 0.8855$ and $Q^{*} = 133.72$, TP_1^{*}= 799.075 (Figure 4).

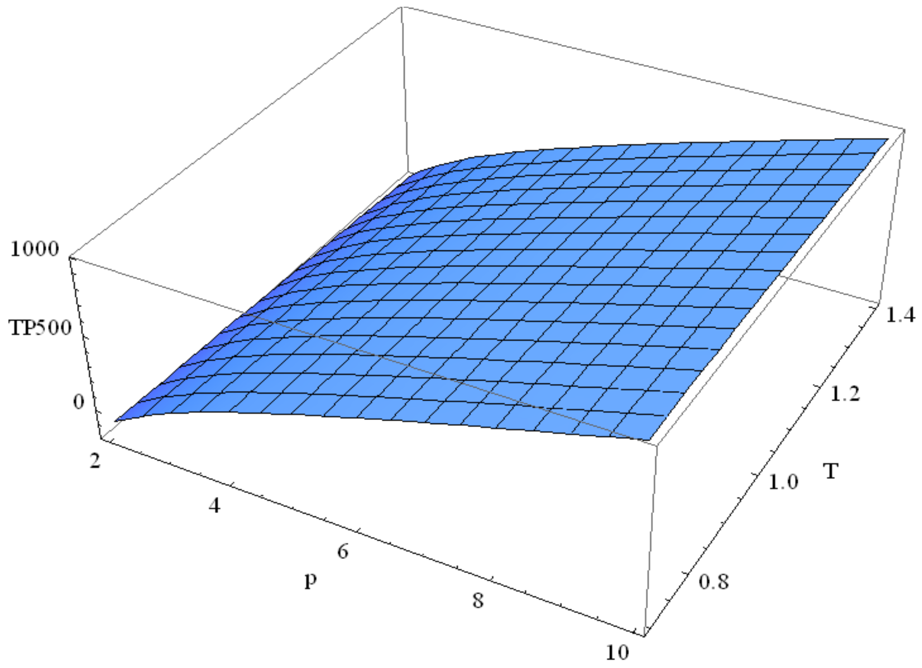

Fig. 4. Concavity of the total profit function (TP_1).

Case I-Sub-case II: $M \leq T < N$

To demonstrate the model, the following inventory parameters in appropriate units:

$A = 20$,	$a = 100$,	$b = 0.4$,	$c = 1.8$,	$q = 1.2$,	$h = 0.2$,
$k = 0.2$,	$\theta = 0.01$,	$\alpha = 40$,	$\beta = 50$,	$c_d = 0.2$,	$r = 0.5$,
$M = 1.5$,	$N = 3$,	$Ic_1 = 0.18$,	$I_e = 0.12$.		

By Eq. (14), we get optimum values of p^{*}= 5.8830, $T^{*} = 1.5$ and $Q^{*} = 248.329$, TP_2^{*}= 600.447 (Figure 5).

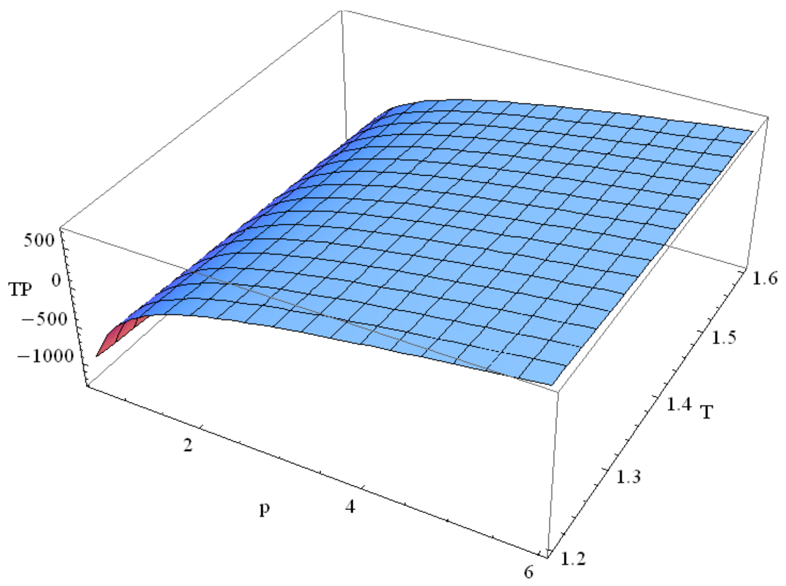

Fig. 5. Concavity of the total profit function (TP_2).

CaseII-Sub-case: $M < N \leq T$

To demonstrate the model the succeeding inventory parameters in appropriate units:

$A = 20$, $a = 100$, $b = 0.4$, $c = 1.8$, $q = 1.2$, $h = 0.2$,
$k = 0.2$, $\theta = 0.01$, $\alpha = 40$, $\beta = 50$, $c_d = 0.2$, $r = 0.5$,
$M = 1.5$, $N = 3$, $Ic_1 = 0.18$, $Ic_2 = 0.28$, $I_e = 0.12$.

By Eq. (27) and solving it, we get optimum values of $p^* = 6.4400$, $T^* = 3$ and $Q'^* = 628.925$, $TP_3^* = 132.825$ (Figure 6).

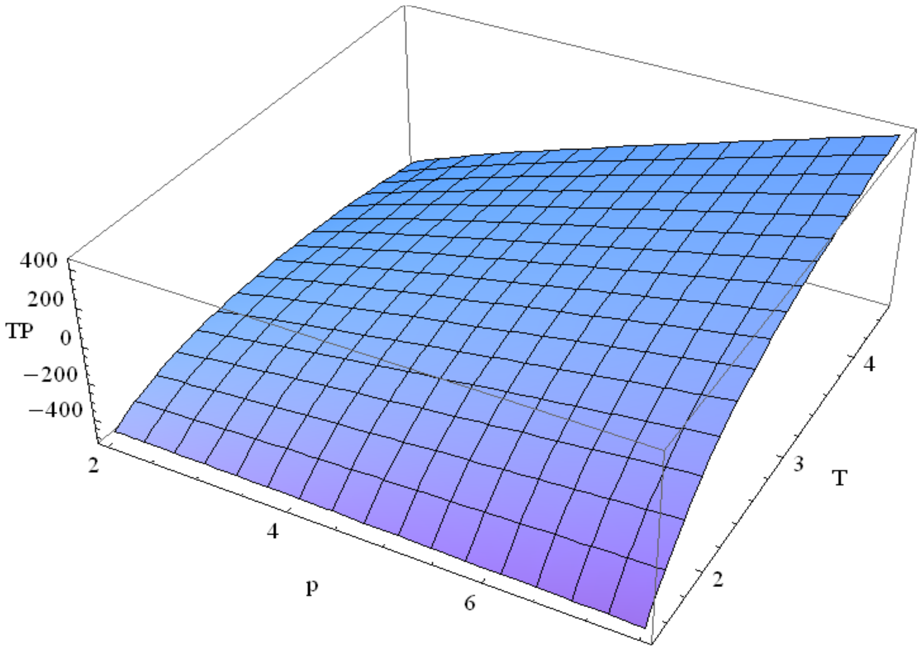

Fig. 6. Total profit function (TP_3).

7. Sensitivity Analysis

Sensitivity analysis provides extra comprehension of different parameters related to inventory which helps in the demonstration of results. For instance, findings related tosmall variations in the associated parameters can provide major variations in the decision variables. Also, in order to examine the strength of the projected framework, it is appropriate to conduct thesensitivity analysis.Further, this examination has been completed by changing the estimation of every parameter from – 30% to 30% in any case, one parameter at any given time with all the remaining parameters holding their underlying qualities. The results obtained from the example, it is observed that sub-case Iis more profitable rather among all sub-cases. On the similar lines, the sensitivity analysis was conducted with the modifications in cost and demand parameters such as deterioration rate, demand rate parameters, holding cost, inflation rate, purchasing cost, ordering cost etc. and exposed the effect on the replenishment cycle time, selling price, and the total profit.

Table 1 and Figures 7-9 display the obtained sensitivity analysis results.

Table 1. Sensitivity analysis of different inventory parameters.

Parameter	% Changes	Change in p^*	Change in T^*	Change in TP^*	Change in Q^*
a	-30	7.9120	0.8963	538.49	104.05
	-20	7.9941	0.8898	614.15	113.55
	-10	8.0203	0.8839	685.77	123.20
	10	8.4211	0.8815	874.89	142.49
	20	8.5660	0.8802	968.42	152.32
	30	8.6780	0.8789	1060.26	162.24
b	-30	7.6541	0.7712	672.51	113.66
	-20	7.8462	0.8015	706.97	118.19
	-10	8.3618	0.8419	779.15	122.96
	10	8.4110	0.9419	816.59	144.72
	20	8.4116	1.0226	834.70	162.85
	30	8.4120	1.0228	835.10	163.01
h	-30	8.4011	0.8959	802.22	134.52
	-20	8.4015	0.8924	801.18	133.89
	-10	8.4018	0.8889	800.13	133.26
	10	8.4024	0.8822	798.04	132.06
	20	8.4028	0.8789	797.02	131.47
	30	8.4033	0.8757	796.01	130.90
c	-30	7.9836	0.9353	832.95	143.39
	-20	8.1717	0.9175	826.93	139.32
	-10	8.3413	0.9016	819.12	135.77
	10	8.5011	0.8716	783.63	129.82
	20	8.7109	0.8608	781.25	127.20
	30	8.9231	0.8511	779.41	124.83
θ	-30	8.4002	0.8835	800.34	132.19
	-20	8.4009	0.8842	799.93	132.35
	-10	8.4011	0.8849	799.46	132.51
	10	8.4028	0.8862	798.67	132.81
	20	8.4035	0.8869	798.27	132.97
	30	8.4044	0.8875	797.89	133.11
A	-30	8.3600	0.8634	801.09	128.87
	-20	8.3643	0.8706	799.28	130.13
	-10	8.3712	0.8779	797.79	131.41
	10	8.4064	0.8927	797.33	133.92
	20	8.4084	0.8999	795.33	135.21
	30	8.4116	0.9070	793.48	136.47

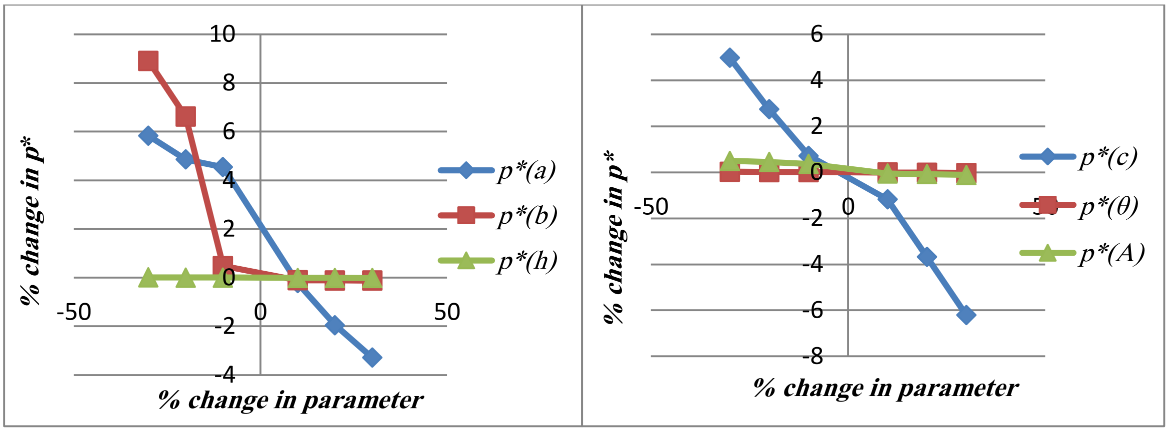

Fig. 7. Variation in selling price by changes in different parameters.

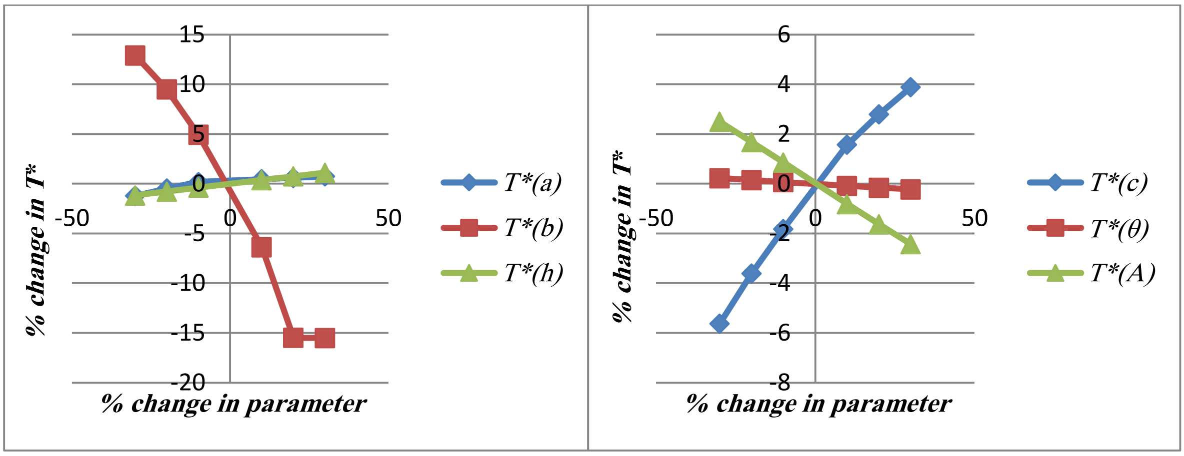

Fig. 8. Variation in total cycle time by changes in different parameters.

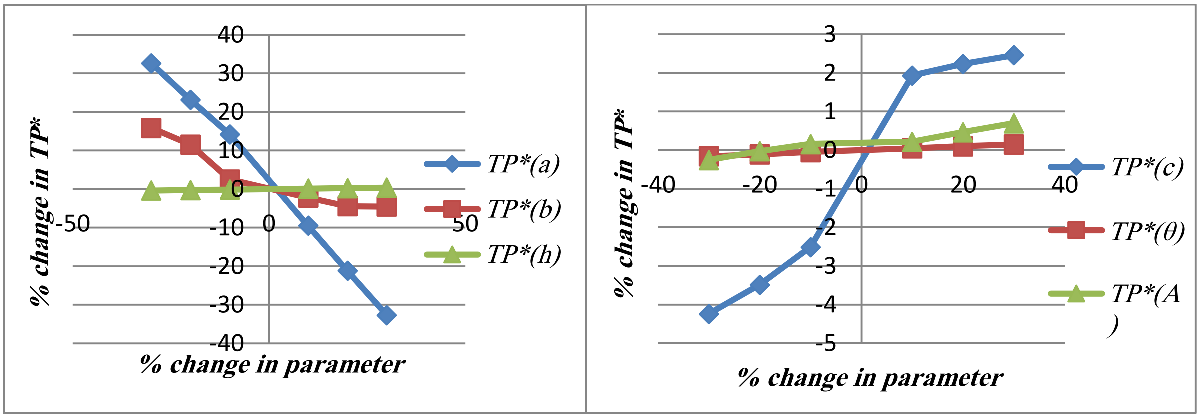

Fig. 9. Variation in total profit by changes in different parameters.

The observed managerial issues are as follows:

- If the units of demand '*a*' increase, then the selling price, optimum order quantity and total net profit increase, but the optimal cycle length decreases. Retailers purchase more items due to increasing demand, and sales revenue costs generate more profit.
- If the parameter '*b*' increases, then the optimum selling price, the optimal cycle length, the optimum order quantity, and the net profit also increase.

- An increase in parameter 'h' lowers the optimum cycle length, order quantity and total net profit. However, the price at which an item is sold increases very leisurely. It is evident that if a retailer holds a huge quantity, the extra cost of holding will reduce the profit.
- An increment in the purchasing cost 'c' parameter reduces the optimum cycle time, order quantity and the total net profit, whereas positive change has been observed in the selling price. Net profit is also reduced if the retailer purchases their items at a high cost.
- As the deterioration rate 'θ' increases while the selling price, order quantity and optimal cycle length slightly increase, the total net profit decreases. It is verifiable truth that the deterioration negatively impacts total profit. So, retailers must request a smaller quantity to avoid excess decay. Thus, total profit decreases as the deterioration rate increases.

- As the ordering cost 'A' is increases, the total net profit decreases simultaneously. It means that the retailer may order less quantity to reduce net profit. The optimal selling price, optimum order quantity, and optimal cycle are increasing concerning increments in A.

8. Conclusion

This paper has presented an EOQ inventory model for deteriorating items, considering that the supplier offers a progressive interest scheme to its retailer in an inflationary environment. The proposed model is solved and analysed through numerical illustrations to optimise the proposed framework's outcome. It is recommended that the retailer should enjoy the granted credit- period by putting in more minor requests, and some credit ought to be passed on to the clients. This advantages retailers by getting records settled before with the assistance of a second credit period and lessening the danger of stock shortness. The result of the study is presented in numerical examples for all the derived cases, and the study's finding suggests that sub-case I has maximum profit when the retailer's credit period is greater than the replenishment period in the context of all discussed cases. This analysis of the model uncovered that the proposed system is vigorous enough underneath the outcomes of referred inventory parameters.

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