International Journal of Applied Optimization Studies
IJAOS
İSÜ İSTİNYE ÜNİVERSİTESİ İSTANBUL

An Innovative Two-Phase Local Search Approach for Efficient University Course Scheduling under Academic Constraints

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ABSTRACT

Scheduling university courses represents a formidable challenge, given the multifaceted academic constraints inherent to higher education institutions. This paper addresses the intricate task of optimizing university course scheduling, which involves accommodating numerous constraints, such as classroom and faculty availability. While numerous methodologies have been proposed to tackle this problem, our work introduces a novel approach employing a two-phase local search method. In the initial phase of our method, a viable solution is generated, laying the foundation for a well-structured course schedule. Subsequently, in the second phase, we refine the generated schedule using carefully crafted techniques to enhance its overall quality. Our approach has been meticulously implemented and rigorously evaluated across various datasets, yielding promising results. The outcomes of our study underscore the efficacy of our two-step methodology in resolving the complex challenge of university course scheduling. This research contributes a robust solution to aid academic institutions in streamlining their course scheduling processes while adhering to academic constraints.

Paper type: *Research Article*

Received 2025-02-18
Revised 2025-04-27
Accepted 2025-05-19

Keywords:
Vehicle routing;
Fleet selection;
Product segmentation;
Electric vehicles;
Delivery;
Metaheuristic algorithm.

1. Introduction

Vehicle routing has emerged as a pivotal concern of global significance in recent decades, with the primary objective of enhancing the efficiency and productivity of transportation systems. Broadly speaking, a vehicle routing problem (VRP) involves the task of servicing a group of delivery customers using a fleet of vehicles stationed at a central depot or Distribution Center (DC) situated within the same geographic region as the customers. The central goal is to leverage mathematical and optimization models to minimize various factors, including travel distance, total travel time, vehicle count, tardiness penalties, and transportation costs, all while simultaneously maximizing customer satisfaction.

Ultimately, the VRP seeks to identify the most efficient route that visits all cities exactly once before returning to the initial starting point.

Each VRP can be broken down into four fundamental components: inventory, the set of customers, the fleet of vehicles, and the road network. Notably, VRP plays a crucial role in supply chain distribution systems, and its optimal resolution significantly reduces distribution expenses, ensures the timely delivery of goods, curbs inventory costs, and elevates customer contentment. The historical context of this problem can be traced back to its formal introduction in 1959 by Dantzig and Ramser (1959). In a noteworthy study, Tavakkoli-Moghaddam et al. (2006) applied a hybrid simulated annealing approach to address capacitated VRPs with independent routes. In another research endeavor, Narasimha et al. (2013) proposed an ant colony optimization technique for addressing the min-max multi-depot vehicle routing problems, with the primary objective of minimizing the longest travel distances.

Adelzadeh et al. (2014) introduced a mathematical model and a heuristic solving procedure for addressing multi-depot VRPs with fuzzy time windows and diverse vehicle types. In a similar vein, Afshar-Nadjafi and Afshar-Nadjafi (2014) devised a constructive heuristic tailored for time-dependent multi-depot VRPs, featuring time windows and a heterogeneous vehicle fleet, all aimed at cost minimization.

Goeke and Schneider (2015) undertook the challenge of modeling the routing of a mixed fleet comprising both electric and conventional vehicles. Their model considered variables such as fuel consumption based on speed, road gradient, and cargo weight. To complement their efforts, they developed a local search optimization technique. Doppstadt et al. (2016) explored methods to reduce carbon emissions through the use of hybrid electric vehicles in short-distance transport. Their solution strategy relied on the Tabu search method.

Furthermore, Hiermann et al. (2016) proposed a model for determining fleet size in an electric and mixed VRP context, which encompassed time windows and recharging stations. Their approach relied on the branch and price method and an extensive neighborhood search strategy.

Keskin and Çatay (2016) crafted a model for electric VRPs with time windows and effectively solved it using an adaptive large-scale neighborhood search (ALSN) algorithm.

Schiffer and Walther (2017) addressed the electric location routing problem, complete with time windows and partial recharging considerations. They introduced a routing location approach that concurrently accounted for the routing of electric vehicles and the strategic placement of chargers.

Zhang et al. (2019) tackled an innovative location-routing problem in electric vehicle transportation, incorporating stochastic demands. Their solution method relied on a VNS-based hybrid algorithm.

Pelletier et al. (2019) delved into the electric vehicle routing problem while accounting for uncertainties in energy consumption. They devised a linear model for smaller dimensions, utilizing robust optimization techniques and a two-step heuristic employing neighborhood search for larger-scale

scenarios. Finally, Macrina et al. (2019) formulated a model for electric vehicle routing involving standard time windows and partial charging. They efficiently addressed this extensive model by employing an iterative local search (ILS) approach.

Building upon previous research in the realm of vehicle routing problems, a significant concern has arisen regarding the segregation of fleets based on the nature of goods being transported. Notably, a research gap exists in evaluating routing challenges through the lens of fleet separation grounded in the specific type of merchandise. To address this gap in the existing body of knowledge, the current study endeavors to bridge this gap.

In the scope of this investigation, the Alborz Distribution Co. emerges as a central player, responsible for the simultaneous distribution of a diverse array of goods. Specifically, the company manages the distribution of both pharmaceutical and consumer products to its clientele. In accordance with the regulations outlined by authoritative bodies such as the Food and Drug Administration, encompassing the principles of Good Distribution Practice (GDP) and Good Storage Practice (GSP), it is strictly forbidden to consolidate the loading and dispatch of these two distinct product categories within a single fleet in distribution firms. Instead, strict compliance necessitates the separation of goods into distinct vehicles designated exclusively for pharmaceutical and consumer products.

This quandary regarding fleet separation bears significant ramifications, particularly concerning the preservation of product integrity, especially in the case of delicate pharmaceutical items. Furthermore, unwavering adherence to the principles of fleet separation holds the potential to enhance customer satisfaction significantly. Consequently, the present research embarks on the task of formulating a routing problem that explicitly takes into account the variability in distributed products, with fleet separation based on the type of goods being transported serving as a pivotal focal point.

2. Problem Description

The Vehicle Routing Problem (VRP) is a critical issue within the complex landscape of supply chain distribution networks. These networks typically encompass fleets with specified capacities, multiple inventories, diverse demand centers, and an intricate network of routes connecting inventories to these demand centers.

In this context, the Alborz Distribution Co. emerges as a prominent player in Iran's distribution sector, specializing in the distribution of pharmaceutical and consumer goods. Due to the fundamental nature of its operations, product distribution assumes utmost significance. Consequently, distribution costs take center stage as the primary operational expense for the organization. The primary objective of our model's objective function is to directly address the organization's paramount concern: the reduction of operational distribution costs.

The assumptions of the proposed mathematical model are as follows:

- The existing fleets are divided into two owned and rental classes.
- Priority of vehicle allocation to tours is based on the cost of each fleet.
- The tours of owned vehicles are closed, meaning that the fleet will return to the inventory after meeting the last customer.
- The tours of rental vehicles are open, meaning that the fleet will not return to the inventory after meeting the last customer.
- All fleets have the maximum time allowed for customer allocation.
- A specific service time is provided for the process of delivering goods to customers per customer.
- The fleet related to pharmaceutical and consumer goods is different, and fleet allocation is based on the type of goods.
- The vehicles in the model are assumed to be heterogeneous. In this regard, fleet heterogeneity includes the following items:
- Fleet weight and volume capacity
- Fleet availability time
- Property and contractual status of the fleet
- The fixed cost of using the fleet
- Status of allocation to pharmaceutical or consumer goods

A pivotal concern when dealing with models encompassing diverse fleets is determining vehicle allocation priorities for tours. In this specific model, the allocation process is contingent on the fixed costs associated with fleet utilization. In essence, the mathematical model initially strives to assign the most cost-effective fleet to existing tours to minimize expenses. Subsequently, it progresses to vehicles incurring higher fixed costs, following a predefined priority scheme. This progression occurs incrementally, contingent upon each fleet's capacity fulfillment.

Furthermore, an analysis of real-world data obtained from the Alborz Distribution Co. reveals instances where the cumulative demand of significant customers exceeds the capacity of the available vehicles. This scenario makes it impossible to fulfill the delivery of goods to customers using a single vehicle. Hence, multiple vehicles must be employed to address this situation. In scholarly literature, this challenge is recognized as the "partial delivery problem."

3. Mathematical Model

Here, we first present the parameters and decision variables and the mathematical model of the problem.

Sets

N: Set of nodes	$N = \{0,1,2,\dots,n,n+1\}$
N_o: Set of customers	$N_o = \{1,2,..,n\}$

N_f : Set of charging stations

Start inventory {0}

End inventory {n + 1}

K :Vehicles

K_r: Rental vehicles

K_0: Owned vehicles

p: Type of goods

Parameters

C_{ij}: Travel cost from the i-th node to the j-th node; $i, j \in N$

d_i: The i-th customer demand; $l \in N_0$

st_i: The service time of the i-th customer; $l \in N_0$

(a_i, b_i): Time interval of the i-th node; $i \in N$

F_c: Cost of using the vehicle k; $k \in K$

cap_k: The capacity of vehicle k; $k \in K$

T_{max}: Maximum time available

t_{ij}: Travel time between the i-th and j-th nodes

w_{ip}: 1, if the demand of the i-th customer is of p-type goods; otherwise, 0.

r: Fuel consumption rate

f_k: Battery capacity of vehicle k; $k \in K$

Decision Variables

X_{ijk}: If vehicle k moves from the i-th node to the j-th node.

S_{ik}: The time of arrival of vehicle k to the i-th node

y_{kp}: 1, if vehicle k carries p-type goods; otherwise, 0.

f_{ik}: A fraction of demand of the i-th customer delivered by vechile K.

w_{ik}: Fuel level of vehicle k when leaving the i-th node

q_{tk}: Fuel level of vehicle k when entering the t-th charging station

Mathematical Model

$$\min \ Z = \sum_{t\in N}\sum_{k\in K} F_0 X_{otk} + \sum_{t\in N}\sum_{j\in N}\sum_{k\in K} C_{ij} X_{ijk} - \sum_{t\in N}\sum_{k\in K} C_{t,n+1} X_{t,n+1,k} \quad (1)$$

$$\sum_{\substack{i\in N \\ l\neq 0}} X_{jik} - \sum_{\substack{i\in N \\ l\neq N+1}} X_{ijk} = 0 \qquad \forall i \in N_0, \ \forall k \in K \quad (2)$$

$$\sum_{k\in K} f_{tk} - 1 \qquad \forall i \in N_0 \quad (3)$$

$$\sum_{l \in N_0} f_{tk} = cap_k \qquad \forall k \in k \tag{4}$$

$$\sum_{j \in N} X_{ijk} \geq f_{tk} \qquad \forall i \in N\{0\} \quad \forall k \in K \tag{5}$$

$$\sum_{t \in N} \sum_{j \in N} (st_i + t_{ij}) X_{ijk} \geq T_{max} \qquad \forall k \in K \tag{6}$$

$$\sum_{t \in N_0} X_{otk} \leq 1 \qquad \forall k \in K \tag{7}$$

$$S_{tk} + st_i + t_{ij} - T_{max}(1 - X_{ijk}) \leq S_{jk} \qquad \forall i, p \in w_{tp}, \forall j \in N, \forall k \in K \tag{8}$$

$$a_t \leq S_{tk} \leq b_t \qquad \forall i \in N, \forall k \in K \tag{9}$$

$$\sum_{p \in P} \gamma_{kp} = 1 \qquad \forall k \in K \tag{10}$$

$$X_{ijk} \leq \gamma_{kp} \qquad \forall i, p \in w_{tp}, \forall j \in N, \forall k \in K \tag{11}$$

$$r, c_{ij}, x_{ijk} - f_k(1 - x_{ijk}) \leq w_{ik} - w_{jk} \leq r, c_{ij}, x_{ijk} + f_k(1 - x_{ijk}) \quad \forall i \in N, j \in N_0, \forall k \in K \tag{12}$$

$$r\ c_{ij}\ x_{ijk} - f_k(1 - x_{ijk}) \leq w_{ik} - q_{ik} \leq r\ \ c_{ij}\ x_{ijk} + f_k(1 - x_{ijk}) \quad \forall i \in N, j \in N_f, \forall k \in K \tag{13}$$

$$w_{ik} \geq r\ C_{t,n+f+1}\ x_{t,n+f+1} \qquad \forall t \in N, \forall k \in K \tag{14}$$

$$w_{tk} = f_k \qquad t = \{0\}, \forall k \in K \tag{15}$$

$$X_{ijk} \in \{0,1\} \qquad \forall i, j \in N, \forall k \in K \tag{16}$$

$$y_{tp} \in \{0,1\} \qquad \forall k \in, \forall p \in P \tag{17}$$

$$S_{tk}, f_{tk} \geq 0 \qquad \forall i \in N, \forall k \in K \tag{18}$$

$$W_{tk}, q_{tk} \geq 0 \qquad \forall t \in N, \forall k \in K \tag{19}$$

Equation (1) constitutes the model's objective function, responsible for computing the initial component of vehicle utilization costs. The second term accounts for the total travel expenses incurred by all vehicles, while the third term subtracts the cost associated with returning rental vehicles to the inventory from the overall costs.

Equation (2) ensures the presence of an output path for each input path related to the customer. In simpler terms, these constraints guarantee path continuity. Constraint (3) mandates the satisfaction of all customer demands, leaving no deliveries unattended. Moreover, Constraint (4) ensures that the cumulative demands of customers assigned to a single vehicle do not exceed its capacity. Constraint (5) addresses the issue of partial deliveries within the model.

Moving forward, Constraint (6) requires that the total time spent by a vehicle and the duration of customer service must not exceed the total available time. Constraint (7) firmly establishes that each vehicle is deployed at most once, avoiding duplications. Constraint (8) calculates the arrival time of each vehicle at every node, effectively eliminating any sub-tour possibilities. Constraint (9) enforces

that the arrival time at each node must fall within the prescribed time window for that specific node. Constraint (10) dictates that each vehicle can transport only one type of product.

Constraint (11) stipulates that vehicle 'k' can serve both the 'i-th' and 'j-th' customers if it carries 'p'-type goods. Constraints (12) and (13) monitor the fuel levels of vehicles upon reaching customers and charging stations. Constraint (14) ensures that the fuel level remains adequate for the return journey to the inventory from the last visited node. Constraint (15) specifies that the battery charge level when departing from the inventory, after visiting the charging station, must equal the battery capacity. Finally, Constraints (16) to (19) pertain to the domain and decision variables.

4. Proposed Method

Given the NP-Hard complexity of the routing problem, as emphasized by Goli et al. in their 2018 work, the utilization of heuristics becomes imperative when addressing such intricate challenges. In this study, we employ the Simulated Annealing (SA) metaheuristic algorithm to tackle the proposed model, particularly in scenarios involving large dimensions.

In essence, SA represents a stochastic approach designed to solve complex, hybrid problems. The primary aim of this algorithm is to randomly select a solution closely related to the current one at each iterative stage. Subsequently, this selected neighboring solution replaces the current solution if it results in a superior objective function. However, if the new solution yields a less favorable objective function, it still has a chance to replace the current one based on a probabilistic assessment, with the probability ranging between zero and one.

In the SA algorithm, the probability of opting for a solution with a less favorable objective function is directly linked to the difference between the objective functions in the solution. Solutions with minimal differences from the current objective function are favored, while those with significant disparities are less likely to be chosen. Consequently, the likelihood of settling for suboptimal local solutions diminishes as the algorithm progresses through iterations. Simultaneously, the algorithm gradually converges towards an optimal solution of high quality, thanks to the reduction in system temperature, as elucidated by Goli et al. (2018).

4.1. SA Structure

The stages of SA are presented below:

Stage 1. Reading the input data of the problem.

Stage 2. Creating a feasible solution z_0 and estimating its corresponding objective function value $f(z_0)$.

Stage 3. Reserving z_0 as the best solution (z_{Best}) and $f(z_0)$ as the best value of the z_{Best} objective function.

Stage 4. Receiving the SA parameters.

Stage 5. Repeating k times (repeating the outer loop).

Stage 6. Repeating per 1 neighbor (repeating the inner loops) and establishing a z' to z neighborhood and calculating *f(z')*.

Stage 7. Estimating $\Delta = f(z') - f_{Best}$, if $\Delta < 0$. Then, $z_{Best} = z'$ and $f_{Best} = f(z')$. Otherwise, if $p = e^{\frac{-\Delta}{T}} > z_{Random}$, then $z_{Best} = z'$.

Stage 8. If fairness z_{Best} does not change in the number of inner iterations, it will be excluded from the loop.

Stage 9. End of iteration 1.

Stage 10. Decrease the current temperature to $T_r = \alpha . T_{r-1}$.

Stage 11. If z_{Best} does not change in the number of outer iterations, it will be excluded from the loop.

Stage 12. End of iteration k.

Stage 13. End of algorithm and solution representation.

4.2. Solution Presentation Method

Consider a scenario where X represents the number of customers, W denotes the count of fuel stations, and Y stands for the quantity of vehicles in the system. The solution chromosome can be envisioned as a permutation involving dimensions equal to (X + W + Y - 1).

In this particular study, we adopt a distinctive approach, utilizing two separate chromosomes for each customer group. This choice is rooted in our assumption of segregating customers and vehicles based on the categories of pharmaceutical and consumer goods. For instance, let's consider a scenario with two vehicles, one owned and one rented, serving six customers with pharmaceutical goods, all located within proximity to two fuel stations.

The architecture of a chromosome takes the form of a 1*9 matrix, where numbers 1 to 6 represent customers, numbers 7 and 8 denote the locations of fuel stations, and number 9 serves as a separator to distinguish between distinct routes.

As illustrated in Figure 1, the owned vehicle embarks on a route that caters to customers 2, 5, and 4, commencing from the depot. Subsequently, it concludes its journey with a visit to the fuel station before returning to the depot. In contrast, on a separate route, the rented vehicle first tends to customers 6 and 1, followed by a stop at the designated fuel station. Eventually, it completes its route by serving customer 3.

2	5	4	7	9	6	1	8	3

Fig. 1. Solution representation method.

4.3. Neighborhood Establishment Mechanism

In order to achieve a new solution in the SA, changes should be made in the structure of the current solution to establish a new neighborhood in the solution space. Three operators are applied in the algorithm to create a neighborhood. In fact, each operator is randomly selected in each iteration of the algorithm and applied to the related solution.

4.3.1. Swap

Nodes 3 and 4 are selected from the Figure 2 chromosome in the swap operator, and they change places.



Fig. 2. Swap operator presentation method.

4.3.2. Reversion

Two nodes 3 and 4 are selected from Figure 3 in the operator, and the reversion of nodes between them is carried out in addition to their replacement.



Fig. 3. Reversion operator presentation method.

4.3.3. Insertion

Nodes 3 and 4 are randomly selected in Figure 4. In addition, node 3 is eliminated and transferred to next to node 4.



Fig. 4. Insertion operator presentation method.

4.4. Parameter Tuning

In the realm of metaheuristics, the performance of algorithms is profoundly influenced by the values assigned to their parameters. Consequently, the precise determination of these parameter values holds paramount importance. Various methodologies have been developed for the fine-tuning of algorithm parameters. In our current research, we have employed the Taguchi method.

The Taguchi method operates by presenting a range of experimental designs to the researcher, each design encompassing different levels for the factors that influence the experiment. The experimenter then conducts experiments by selecting an appropriate design suggested by the Taguchi method. Subsequently, the Taguchi method facilitates an analysis to determine the impact of each factor on the dependent variable (Goli et al., 2021).

In our parameter tuning process, we focused on three key parameters: the number of iterations (MaxIt), initial temperature (T0), and the refrigeration coefficient (alpha). We considered three distinct levels for each of these parameters. Consequently, a series of experiments were meticulously conducted to facilitate Taguchi analysis, utilizing Minitab version 17. The parameter level domains can be found in Table 1, and Figure 5 graphically illustrates the varying levels of change in the S/N index. Additionally, Table 2 presents the effectiveness levels of each factor.

Table 1. Levels of SA parameters.

Parameters	Low bound	Median value	Upper bound
MaxIt	300	700	1000
T0	50	60	70
alpha	0.85	0.95	0.99

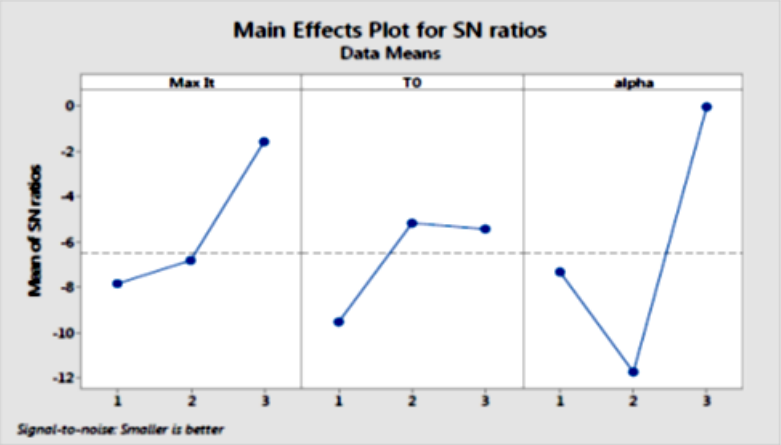


Fig. 5. Changes in values of the S/N index at various SA levels.

Table 2. Optimal levels of SA parameters.

Parameters	Optimal Value
MaxIt	1000
T0	60
alpha	0.99

5. Numerical Results

In our study, we executed the model using GAMS 23.4.3 to ensure precision and reliability. Figure 6 provides a visual representation of a GAMS problem instance. In this particular example, we observe the operation of two owned and two rental vehicles, serving a total of six customers, including two pharmaceutical and four consumer goods customers, distributed across three distinct routes. It is noteworthy that certain fuel stations may remain unvisited during the course of travel.

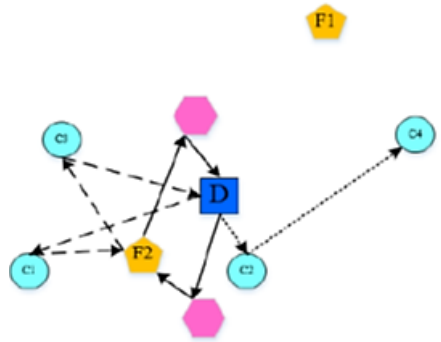


Fig. 6. Schematic presentation of the mathematical model.

To address the mathematical model comprehensively, we examined 30 sample problems. These sample problems were categorized into two modes: one involving fleet separation for consumer and pharmaceutical customers, and the other with no such separation. Table 3 offers an overview of the parameter values employed in the model.

The computational results derived from these sample problems, employing the GAMS software and the SA algorithm, are summarized in Table 4. This table also presents the algorithm's deviation from the exact solution, shedding light on the effectiveness of our approach. As indicated in Table 4, the results highlight that the Simulated Annealing (SA) algorithm performs on par with the solutions generated by GAMS for very small-scale problems. Furthermore, it demonstrates an impressively minimal optimization gap in smaller scenarios, underscoring the algorithm's exceptional performance in achieving optimal solutions. However, as we transition to moderate and large-scale problems, it becomes evident that the GAMS software struggles to reach an optimal solution. It is worth noting that the solution time experiences an exponential increase as the number of nodes grows. Figure 7 offers a comparative analysis of the objective functions produced by GAMS and SA across nine distinct problems. This comparison reveals that the solutions align closely in smaller dimensions, while the optimization gap issue becomes more pronounced with increasing dimensions.

Table 3. Values of model's parameters.

Parameters	Values
N	5,10,15,20,40,70,80,100,110,120,150
C_{ij}	uniform (100, 300)
d_i	uniform (50, 150)
F_0	uniform (200, 600)
cap_k	uniform (100, 300)
T_{max}	500, 100
r	uniform (0.5, 0.9)
f_k	uniform (100, 400)

Table 4. Computational results obtained from solving the sample problems.

Test Problem	GAMS in lack of fleet separation mode	GAMS in fleet separation mode	GAMS time (sec)	SA	SA time (sec)	RPD
1	682.39	760.84	0.21	717.03	7.87	0
2	824.71	934.16	0.25	928.13	12.44	0
3	908.84	1064.18	0.21	1130.26	11.75	0
4	1035.13	1304.85	0.21	1250.54	13.74	0
5	1988.99	2047.04	375.07	2130.53	28.57	0.62
6	2119.12	2528.55	452.77	2527.78	29.92	1.66
7	2115.04	2455.20	559.94	2680.41	35.31	3.76
8	2432.95	3166.80	910.24	3398.72	35.79	9.91
9	2676.53	3051.33	2289.40	3432.26	53.19	12.99
10	-	-	-	3771.45	46.76	-
11	-	-	-	4334.28	43.98	-
12	-	-	-	4974.17	81.15	-
13	-	-	-	5477.97	107.79	-
14	-	-	-	5950.35	104.95	-
15	-	-	-	6969.96	122.55	-
16	-	-	-	8278.87	133.04	-
17	-	-	-	10271.81	193.15	-
18	-	-	-	12416.15	199.11	-
19	-	-	-	11783.57	242.38	-
20	-	-	-	13723.69	300.90	-
21	-	-	-	14148.27	356.42	-
22	-	-	-	14987.57	398.31	-
23	-	-	-	18267.59	543.68	-
24	-	-	-	19064.82	521.08	-
25	-	-	-	21975.83	623.87	-
26	-	-	-	19574.32	571.40	-
27	-	-	-	22509.38	684.53	-
28	-	-	-	23961.15	754.71	-
29	-	-	-	22620.58	704.99	-
30	-	-	-	23169.81	894.11	-

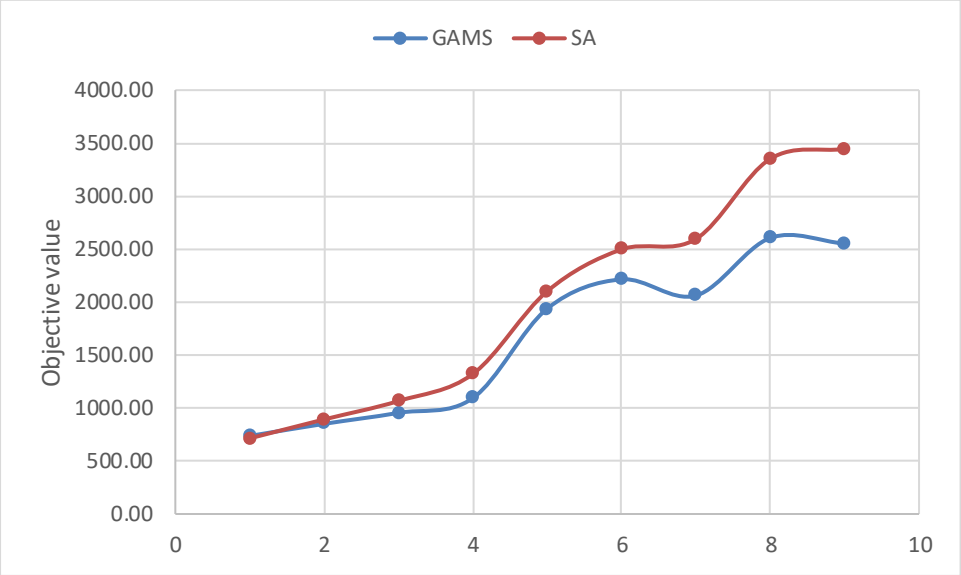


Fig. 7. Objective function values of GAMS and SA.

5.1. Statistical Evaluation of Heuristic and GAMS Software

In this study, ANOVA is used to determine the significant difference between the performance of SA and GAMS software. Table 5 shows the ANOVA results for two criteria of objective function and time of SA and GAMS software. It is worth mentioning that a P-value of less than 0.05 is indicative of a significant difference between the solutions of the two algorithms.

A P-value of 0.839 indicates a notable absence of a significant difference between SA and the GAMS software concerning the objective function criterion. Moreover, a P-value of 0.070 underscores the insignificance of the disparity between the algorithms in terms of computational time.

Furthermore, to better understand their performance, we have constructed 95% confidence intervals concurrently for both algorithms. Figures 8 and 9 visually represent these intervals, affirming the lack of any substantial difference between GAMS and SA with respect to the objective function criterion and computation time, respectively. Notably, SA consistently provides solutions that are remarkably close to the optimal.

Table 5. Analysis of variance for the objective function criterion and computational time.

Criterion	Source	DF	SS	MS	F	P-Value
Objective function	Factor	1	47094	47094	0.04	0.839
	Error	16	17619835	1101240		
	Total	17	17666929			
CPU time	Factor	1	1207680	1207680	3.78	0.070
	Error	16	5106526	319158		
	Total	17	6314206			

Additionally, Figures 8-9 corroborate these findings by illustrating that, on average, GAMS exhibits a higher mean for this particular criterion when compared to SA.

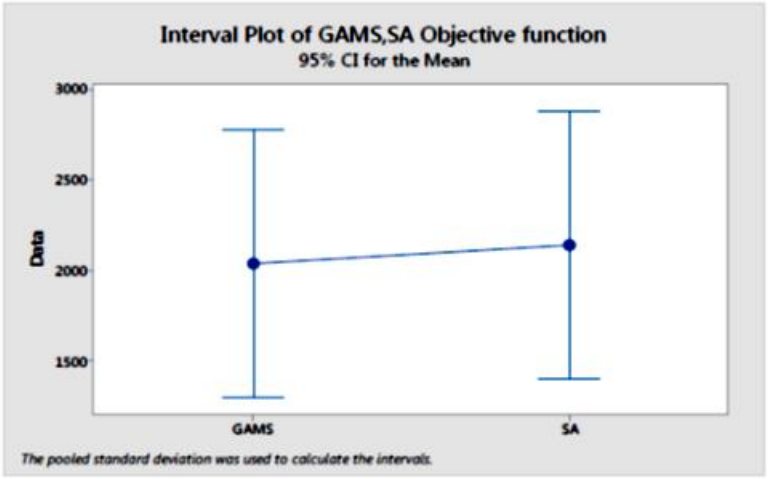


Fig. 8. Diagram of 95% confidence interval of the objective function.

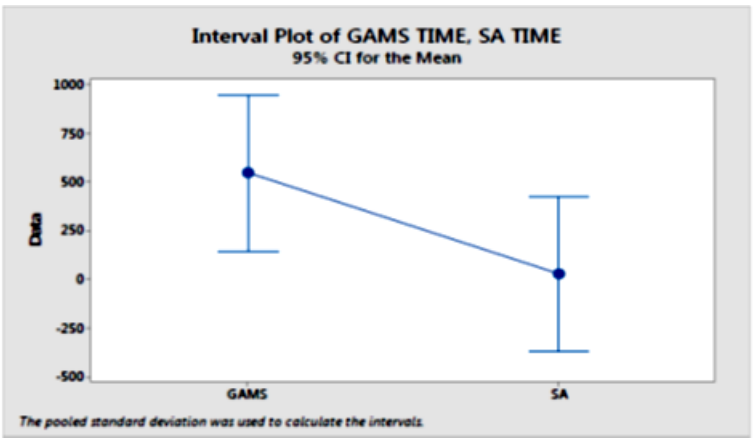


Fig. 9. Diagram of 95% confidence interval of computational time.

6. Conclusion

In the fiercely competitive landscape of today's business world, distribution companies are driven by a paramount goal: cost reduction. Within this financial context, one key expense holds immense significance - the transportation cost incurred in the distribution network. It can be cogently argued that optimizing the transportation system and vehicle routing, especially on a large scale, offers invaluable services in this regard.

Our study introduces a model with an objective function geared towards the minimization of transportation costs. Notably, we direct our focus toward a critical aspect of the transportation system: the separation and categorization of fleets based on the type of goods being transported. This facet holds particular importance for distribution companies responsible for delivering a wide array of goods to

their customers. Furthermore, this model promotes environmental responsibility by incorporating the use of electric vehicles, contributing to pollution reduction.

The model's precision was rigorously validated through its implementation in the GAMS software, followed by the successful resolution of several sample problems employing the SA algorithm, demonstrating its efficiency at smaller scales.

In addition, we conducted a sensitivity analysis, scrutinizing the impact of two vital parameters: fuel capacity and fleet capacity. Our findings underscore the significant role these parameters play in driving down network costs.

A noteworthy outcome, gleaned from the model and our comparative analysis of fleet separation versus non-separation with respect to goods types, reveals that while the quantitative objective function indicates increased costs when vehicles are separated, this approach guarantees the quality of goods. This aspect holds particular relevance for goods with heightened sensitivity, such as pharmaceuticals.

Future research endeavors could delve into topics like developing a multi-objective model, reducing maximum transportation times in the network, minimizing customer wait times, and enhancing overall customer satisfaction as secondary objective functions. Further exploration may also consider evaluating the number of vehicles and travel times probabilistically, in addition to addressing the problem in a multi-depot context.

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