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Game Theory-Based Mathematical Planning Model for Integrated Production Planning in Supply Chains

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ABSTRACT

In the face of intensifying global competition, the efficient management of supply chains has become paramount across practical industries. The intricate nature of supply chain processes, often compounded by extensive computational requirements, presents significant challenges in achieving effective Supply Chain Management (SCM). This article proposes an innovative approach to integrated production planning (IPP), aiming to address these challenges. Inspired by game theory, widely acknowledged for its applications in amalgamating heat and mass converter networks, the proposed approach seamlessly integrates demand and supply data. This integration offers supply chain planners deeper insights into the SCM process, facilitating streamlined reprogramming and decision-making. The framework is substantiated through the presentation of two illustrative case studies. The first involves a single-product scenario, while the second showcases the processing of multiple products on a single processor. Notably, the results obtained from these case studies align consistently with solutions derived from equivalent optimization problems solved using the GAMS. Furthermore, this article introduces a production sequencing algorithm tailored to the second case study. The novel algorithm significantly reduces computation time, achieving a remarkable one-sixth reduction compared to prior methodologies. This reduction underscores the potential for heightened computational efficiency within the proposed framework. In conclusion, the findings of this study demonstrate the efficacy of the game theory-based approach in yielding optimal comprehensive plans and serving as a promising foundation for enhancing computational efficiency through the adept application of mixed programming formulas.

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1. Introduction

1.1. Supply chain management

The manufacturing sector plays a vital role in boosting the economy worldwide through the creation of value and the distribution of goods. All manufacturing businesses share the important task of overseeing material flow, ensuring a seamless journey from suppliers to customers, and adding value through various processes and distribution channels (Stevens, 1989). Over time, different definitions of the supply chain have emerged in scholarly works. It can be understood as a combination of three or more entities, such as organizations or individuals, engaged in both upward and downward flows of products, services, finances, and information (Beamon, 1998). This engagement spans from suppliers to customers and occurs in two ways: the upward stream for acquiring goods/services and the downward stream for sharing information. With the introduction of this fundamental concept in the supply chain realm, the industry turned its focus to effectively managing supply chains. This evolution gave rise to the field of SCM. Modern research definitions of SCM emphasize its role as a comprehensive philosophy aimed at supervising the entire progression of a distribution channel, from suppliers to end users (Cooper et al., 1997).

1.2. IPP in Supply Chain

The intricate interplay between production, inventory management, and distribution within the supply chain underscores the significance of effective coordination and integration among these components (Kumar et al., 2020). In this context, the concept of IPP has gained substantial attention as a strategic approach to optimize supply chain performance (Wang et al., 2020). In general, IPP is a dynamic strategy that holds the promise of harmonizing disparate production elements to foster holistic optimization and operational excellence (Utama et al., 2022).

IPP represents a departure from conventional linear planning models, which often lead to isolated decision-making and inefficiencies across various production stages (Han et al., 2020; Santander et al., 2022). This approach, offers a holistic framework that aligns production processes with broader supply chain objectives, leading to improved responsiveness, reduced lead times, and optimized resource utilization (Denkena et al., 2019).

1.3. Game Theory-based Approaches For Ipp in Supply Chain

As mentioned above, IPP, a fundamental pillar of SCM, is intricately woven with the optimization of resources, costs, and processes across multiple interconnected stages (Pasha et al., 2021). As industries grapple with complexities arising from global markets, varying customer demands, and intricate network structures, the need for sophisticated decision-making frameworks has never been more apparent. Simultaneously, game theory has emerged as a robust tool for modeling and analyzing interactive decision scenarios involving multiple stakeholders (Davis, 2022; Tang et al., 2019). Rooted

in mathematics and economics, game theory provides a structured approach to understanding the strategic interactions between agents, revealing strategies that lead to equilibrium outcomes. The application of game theory in supply chain contexts has gained traction due to its ability to capture the strategic interplay among manufacturers, suppliers, distributors, and retailers, among others (Cachon et al., 2004; Leng et al., 2005).

This paper embarks on a journey to explore the synergistic amalgamation of IPP and game theory in the realm of supply chain management. By delving into the nexus of these two fields, we aim to unravel novel insights and methodologies that empower decision-makers to navigate the intricate web of production, distribution, and coordination challenges. Through a synthesis of theoretical exploration, empirical analysis, and real-world case studies, this article aims to contribute to the burgeoning body of knowledge at the intersection of IPP and game-theoretic approaches.

The subsequent sections of this article are organized as follows: In "Literature review", a comprehensive literature review on the game theory approaches for IPP in the supply chain is provided. Next, in "Methodology", the proposed mathematical model is presented for single and multiple products and finally, in "Conclusions", the research is concluded.

2. Literature Review

The intricate task of optimizing production processes within supply chain networks is acknowledged as a multifaceted challenge, demanding a multidisciplinary approach. Recently, integrating game theory principles into mathematical planning models has exhibited the potential to address intricate strategic interactions and decision-making dynamics among supply chain participants. This section provides a comprehensive literature review, aiming to illuminate the ongoing research on integrated production planning in supply chains and its amalgamation with game theory. The goal is to highlight advancements, identify gaps, and uncover opportunities within this interdisciplinary domain.

In the first decade of this century, several researchers have employed principles of game theory to investigate collaborative and competitive dynamics in the field of chemical engineering supply chains. Gjerdrum et al. (2001;2002) for instance, analyze the equitable distribution of profits among participants in the supply chain using concepts such as transfer prices and Nash bargaining equilibria (Gjerdrum, 2001; Gjerdrum et al., 2002). Aliev et al. (2007) introduced a genetic algorithm-based solution rooted in fuzzy programming to address aggregate production distribution planning within a supply chain context. Hennet et al. (2010) presented compelling insights into the equitable distribution of profits within the context of forming a supply network.

Bai et al. (2012) employed a bilevel formulation to address a competitive dilemma involving the location and planning of a biofuel refinery. This problem is approached as a Stackelberg game, wherein the biofuel refiner assumes the leadership position, followed by farmers who adjust their land use

patterns in response. Chen and Hao (2015) examine the most advantageous pricing and production levels in the presence of emission taxes for rival firms and explore the impact of this tax on the decision-making of these firms. Tominac and Mahalec (2017) critically assess game-theoretic approaches employed in the non-linear, non-convex model within the context of petroleum refinery strategic production planning.

In light of the existing literature, this research introduces several novel and valuable contributions to the domain of integrated production planning within supply chains, augmented by the application of game theory principles. Firstly, by drawing inspiration from game theory, we propose an innovative approach to integrating heat and mass exchanger networks, paving the way for enhanced process efficiency. Secondly, our work emphasizes the significance of achieving a seamless fusion of demand and supply data, revealing deeper insights into the supply chain management process and streamlining re-planning procedures.

Notably, our contributions extend to the practical realm as we present an IPP framework through the exposition of two comprehensive case studies. The first scenario centered on a single product, and the second, intricately addressing the processing of multiple products within a singular processor, showcases the versatility and effectiveness of our proposed framework.

Additionally, we introduce a purpose-built production sequence algorithm tailored specifically to the complexities of the latter case study. These collective contributions underscore the innovative strides our research makes in advancing the understanding and practical implementation of IPP strategies within intricate supply chain contexts.

3. Methodology

This section delineates the systematic approach adopted to develop, implement, and evaluate the game theory-based mathematical planning model for integrated production planning within supply chains. The methodology is structured into several stages, each contributing to the comprehensive analysis of the proposed model's efficacy. In order to showcase the utmost effectiveness of this approach, the methodology will be applied to explore integrated production planning within the supply chain across both single-product and multi-product scenarios. Subsequently, a comprehensive analysis and thorough discussion of the outcomes will be conducted.

As delineated earlier, the primary objective of this paper is to introduce a game theory approach within a comprehensive framework for integrated production planning. This framework is devised to optimize supply chains efficiently. The core principle underpinning integrated planning, aimed at maximizing profits over a defined time span, entails a specific mechanism: it involves fulfilling adequate demand to achieve maximal financial benefits.

In the realm of chemical engineering, game theory has demonstrated significant utility, offering valuable insights for optimizing various resources, such as energy and water. In this research field, the term "pinch" denotes the critical bottleneck within a process. The methodology we propose aligns with this concept and introduces a comprehensive design approach that integrates pinch analysis through graphical representation. This approach serves to delineate objectives and forecast optimal performance, founded on fundamental principles, prior to the actual implementation of process planning (Arshinder et al., 2008).

3.1. Game theory - Chemistry - Supply Chain

The potency of game theory is evident in its capacity to unveil concealed insights, exemplified through the creation of a graph that juxtaposes "Quality" against "Quantity." In the context of heat retrieval systems, quality aligns with temperature, while its quantity denotes the heat duty, inclusive of its enthalpy content. In the realm of mass transfer systems, quality is synonymous with concentration, while quantity embodies mass load or mass flow rate. Within the domain of supply chains, quality is tied to time, while quantity corresponds to the quantity of material (mass, volume, or unit count). This temporal analogy proves particularly impactful when accompanied by a visually intuitive graphical representation and a relatively straightforward interpretation (Cachon, 2003).

In the context of SCM, three vital metrics encompass the flow of materials: material flow rate, material retention levels, and time. Game theory provides an effective framework for addressing these parameters by depicting demand and production trends concerning material quantities (Cachon et al., 2005). In the context of a crucial cumulative planning framework catering to service demand within the time interval $\Delta t = t_k - t_{k-1}$ several decision variables include (Table 1):

Table 1. Required decision variables during the cumulative planning framework catering to service demand.

Decision variable	Description
P_k	The total production output (measured in units) within a time interval t_k
C_k	The total count of externally contracted units (subcontracting) within a time interval t_k
D_k	The accumulated quantity of units requested at time t_k, as per the demand projection
I_k	Inventory level within a time interval t_k
p_k	The amount of production (for example, domestic production within the timeframe $t_{k-1} \leq t < t_k$ at time Δt
c_k	Amount deposited within the timeframe $t_{k-1} \leq t < t_k$
d_k	Demand rate (for example, demand within the timeframe $t_{k-1} \leq t < t_k$ at time Δt) according to the demand forecast

At a specific stage in the supply chain, the fundamental equation governing material flow at time t_k, with I_0 denoting the initial inventory, can be expressed as shown in Eq. (1):

$$I_0 + P_k + C_k = D_k + I_k. \quad (1)$$

Likewise, the equilibrium equation of the substance within the time interval Δt is depicted as presented in Eq. (2):

$$I_{k-1} + P_k \Delta t + C_k = d_k \Delta t + I_k. \quad (2)$$

It is important to note that inventory and stocks cannot exist concurrently within the same time frame. Consequently, stocks can be interpreted as a form of negative inventory. Figure 1 demonstrates that Eq. (2) can be elucidated using traditional combination curves commonly utilized in game theory (Carlsson et al., 2001).

Some prominent characteristics of combination curves are mentioned below.

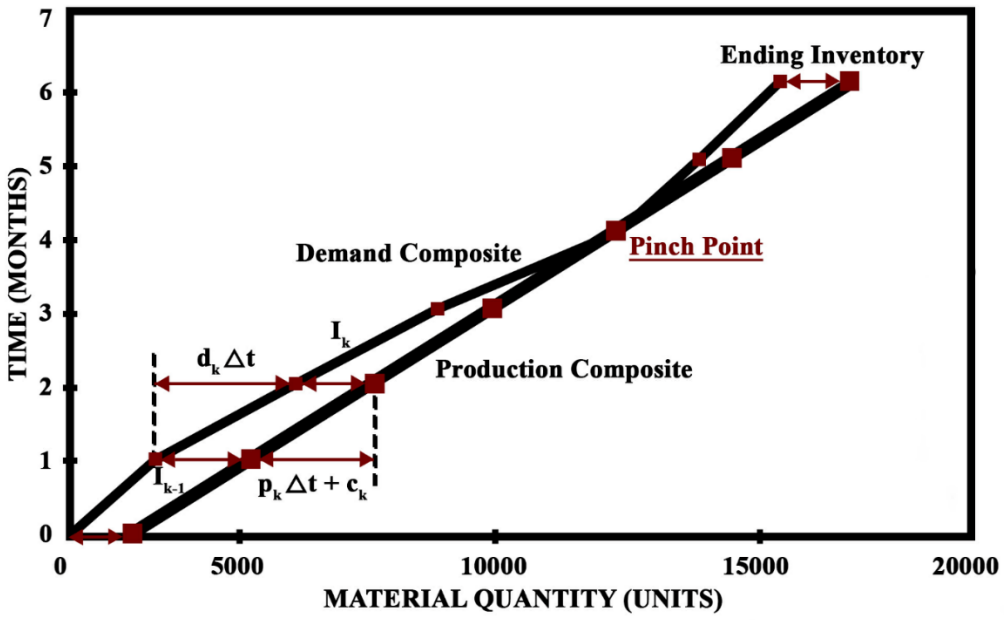


Fig. 1. Game theory in the material (units) - time diagram.

- ✓ The collective demand curve, denoted as D_t, can be easily conceptualized as an accumulating demand function that evolves over time, and it should be in harmony with a composite supply curve labeled as P_t.
- ✓ This demand needs to be met through product supply, with a portion sourced from domestic production and the remainder obtained through outsourcing.
- ✓ This concept is rooted in the fundamental principle of material balance.
- ✓ The crucial factor resides in the vertical variance between the demand and supply composites.
- ✓ In this context, we focus on the time duration between order generation and meeting the demand. In a broader perspective, this timeframe can encompass diverse elements of the supply chain, such

as procurement, manufacturing, and distribution, as well as activities like processing and transportation.

- ✓ There exists a lower limit for the driving force.
- ✓ The specific point where $P_{t-T} = D_t$ is located.
- ✓ These two composites, with minimal time lag, diverge at this critical point.
- ✓ From this perspective, the minimum required time corresponds to the minimum driving force temperature Δt_{min} in heat recovery systems.
- ✓ When $T = 0$, the pinch becomes a point where $P_t = D_t$ (Chang et al., 2006).
- ✓ As we explore these factors, It is crucial to emphasize that the horizontal separation between the two composites at any given moment represents the total inventory within the system.
- ✓ This inventory encompasses work in process (WIP) as well.
- ✓ The pinch is characterized as the juncture at which inventory reaches its lowest level.
- ✓ To be exact, the space between the two composites represents the quantification of inventory levels within the system. When this measure is multiplied by the holding cost factor, it results in the actual inventory expense.

3.2. Single Product Scenario Planning

We have demonstrated the cost breakdown analysis approach in a scenario involving a solitary product. We have used data from a practical case study as an illustration. The product's demand exhibits a seasonal pattern, and the company faces several options, including workforce management (hiring and layoffs), outsourcing specific tasks, and managing inventory or work backlog. The product is typically priced at $100,000 per unit, with a planned $25 per unit discount in March. You can refer to Table 2 for the demand forecast.

Table 2. Demand data for a single product case study.

Period (k)	Month	Forecasted Demand (Units)
1	January	1500
2	February	2800
3	March	2900
4	April	4780
5	May	1520
6	June	1490

The production capacity is primarily determined by the size of the workforce rather than machine capabilities. At the beginning of January, there is an initial stock of 1000 units, and the workforce consists of 80 employees. The factory operates for 20 working days each month, with each employee working an 8-hour shift per day and a maximum of 10 hours of overtime per worker per month. Manufacturing a single unit requires 4 hours of labor. To prioritize exceptional customer service, the

overarching goal is not only to fulfill the entire demand but also to maintain a minimum closing inventory of 500 units by the end of the month of April (according to the solar calendar: Farvardin). Detailed cost-related information is available in Table 3.

Table 3. Cost Information for a single product case study.

Item	Cost (in 100$)
Cost of materials (C_p)	260 units (per each unit)
Cost of inventory units (C_t)	80 units (per unit in a month)
Accumulative lack of inventory penalty (C_s)	150 units (per unit in a month)
Hiring cost (C_h)	8200 unit (per worker)
Warehouse cost (C_l)	13100 units (per worker)
Usual time cost	60 units (per hour)
Overtime cost (C_o)	90 units (per hour)
Subcontracting cost (C_c)	850 units (per unit)

3.2.1. Initial Analysis

The production cost for a single unit during standard working hours amounts to $625, exceeding the subcontracting cost of $750, rendering overtime unnecessary. It is noteworthy to emphasize that the cost associated with holding inventory is $50,000 per unit per month, implying that inventory should not be retained for more than two months. The demand exhibits a consistent rise from month 0 to 4, followed by a subsequent decline from months 4 to 6. Given the considerable expenses tied to fluctuations in capacity (hiring and firing), it is recommended to maintain a consistent capacity approach (strategic level). It is viable to terminate a worker at any point, taking into account the continuous employment wage of $14,500 per month, while the cost of layoff is $12,000.

3.2.2. Initial Summary Plan without Stock Sale

The aggregate demand, as illustrated in Figure 2, is calculated based on the cumulative demand information extracted from Table 2. In cases where the minimum lead time is not specified, it is assumed to be zero. This implies that in the absence of supply, the combined demand serves as an upper boundary for achievable composite production.

To determine the minimal production rate without holding any inventory, the starting inventory of 1000 units is utilized as a pivotal reference point. A fixed production line is adjusted until it precisely intersects with the cumulative demand curve. The coordinates (4, 860 12) mark the critical point. This methodology is reminiscent of the approach introduced by Wang and Smith (1994) for identifying the minimum freshwater pinch, which involves rotating a water supply line.

The compound slope variable represents a minimum production rate of 2965 units per month during the initial 4 months (computed as (4 / (1000-860 12)). Subsequently, owing to a decrease in demand in the following period, it becomes necessary to maintain a closing inventory of 500 units. To meet the

requirements of this phase, the composite profile originating from the pivotal point (4, 860 12) is adjusted to intersect with the endpoint (6, 880 16). This adjustment results in a production rate of 2010 units per month for the final 2 months.

The adoption of a consistent human workforce arrangement combined with minimal inventory, as observed in the continuous composite production, closely mirrors the trend in cumulative demand. Guided by the provided cost data, the outlined production strategy emerges as optimal and seamlessly aligns with the solution derived from solving a linear programming (LP) formulation. Further validation for maintaining a stable human workforce for the initial 4 months, in response to heightened demand, is strengthened through an analysis of the proposed dynamic solution rooted in Kuhn-Tucker conditions (2011), thereby reinforcing the optimality of the solution.

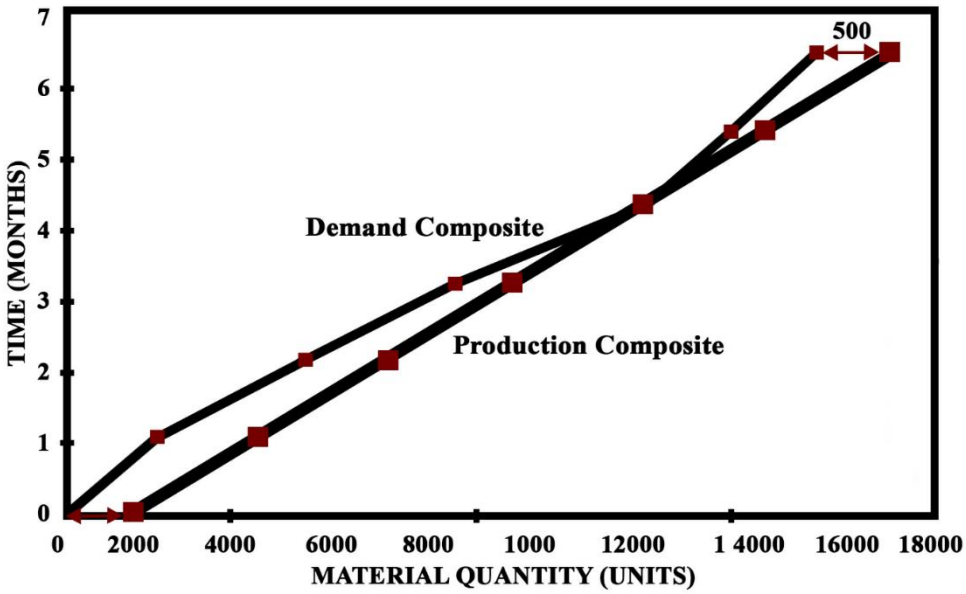


Fig. 2. Composite demand and production in the material (units) - time diagram.

3.2.3. Initial Summary Plan with Stock Sale

Inventory storage leads to delayed customer deliveries but offers the benefit of lowered inventory costs. Let's delve into the process of optimizing the minimal production threshold while considering inventory provision. This ratio represents the slope of the steepest line, which starts from the initiation of inventory and reaches the total cumulative demand, including the required closing inventory.

As a result, the composite production yields a linear trajectory passing through points (0, 1000) and (6, 880 16). Given this consideration, by employing a minimum production quantity of 2,646.67 units per month (e.g., 6.880 15), as referenced in Eq. (2), this approach is outlined in Table 3, which also provides the distribution of allocations. This presentation aligns with the LP solution proposed by previous researchers. The solution's optimality can be confirmed through sensitivity analysis. Moreover,

Kuhn-Tucker's convex conditions (2011) can be utilized to validate the optimality of the solution within the LP model.

Presented here are the particulars of the MILP/LP framework tailored for addressing the extensive unit production planning issue. The central performance goal is directed towards the optimization of total profits across the entirety of the planning timeframe (Bao et al., 2013).

$$\max profit = \sum C_D d_k - \sum C_C c_k - \sum C_P p_k - \sum C_O O_k - \sum C_W W_k - \sum C_H H_k - \sum C_L L_k - \sum C_I I_k - \sum C_S S_k \quad (3)$$

This depicts the accumulation over the entirety of time intervals $(K, \ldots, k = 1)$. It is crucial to note that CD denotes the unit price at which the product is sold, whereas CW represents the continuous wages distributed to workers during each time period. The continuous cost coefficients have been clearly outlined in Table 3. In addition to the decision variables discussed in Section 3.1, Table 4 introduces the subsequent auxiliary variables.

Table 4. Auxiliary variables.

Decision variable	Description
O_k	The cumulative overtime hours within the time interval $t_{k-1} \leq t < t_k$
W_k	Labor at time t_k
H_k	The quantity of labor force engaged within the time interval t_{k-1}
L_k	The count of employees terminated within the time interval t_{k-1}
S_k	Stock (deferred) within the time interval t_k

As such, the nine elements present on the Right-Hand-Side (RHS) of Eq. (3) align with revenue, expenses for outsourcing, material expenditures, overtime expenditures, regular labor outlays, recruitment expenditures, reduction in workforce costs, expenses associated with holding inventory, and stock-related expenditures (Sangaiah et al., 2020). The restrictions concerning the equilibrium of material quantities, labor resources, capacity, and overtime are as follows:

$$I_{k-1} - S_{k-1} + p_k + c_k = d_k + I_k - S_k \quad k \in K, \quad (4)$$

$$W_k = W_{k-1} + H_k - L_k \quad k \in K, \quad (5)$$

$$p_k \leq N_1 W_k + \frac{O_k}{T_1} \quad k \in K, \quad (6)$$

$$O_k \leq T_o W_k \quad k \in K. \quad (7)$$

In this context, N_1 represents the number of units a worker can produce within each standard production timeframe. T_1 denotes the duration required to manufacture one unit, including the maximum allowable overtime hours a worker can participate in during each time interval. Furthermore, K refers to the set of time periods spanning the planning horizon.

At the beginning of the planning horizon, the workforce and the initial inventory/backlog are determined, as well as the inventory/backlog status at the end of the planning horizon. In cases where storage is not utilized, $S_k = 0$. If the number of workers, represented as W_k, is considered an integer variable, the model takes on the characteristics of a Mixed-Integer Linear Programming (MILP) framework. Alternatively, it functions as a Linear Programming (LP) model when W_k is handled differently.

Table 5. Initial summary plan.

Period (k)	Cumulative demand (D_k)	Production (P_k)	Out of source (C_k)	Inventory (I_k)	Slack (S_k)	Worker (W_k)	Number of unemployed
0	0	0	0	1150	0	90	0
1	1500	2500	0	2150	0	70	15
2	4400	2500	0	1550	0	70	0
3	7600	2500	0	990	0	70	0
4	13230	2500	0	0	1350	70	0
5	14950	2500	0	0	320	70	0
6	17020	2500	0	400	0	70	0
7	16250	4000	0	260	200	6250	175

3.2.4. Final Plans

In Table 5, the workforce comprises 66,167 personnel, a number that can be rounded to the nearest whole value. Over the course of 6 months, a team of 66 employees is accountable for producing 15,840 units, which falls short of the total required production by 40 units. These additional 40 units are intended to be outsourced during the month of July, synchronizing with the inventory stocking phase. This scenario of external sourcing is visually depicted as a flat, horizontal line segment within the production composite.

The final strategy, formulated on the basis of the earlier rationale and utilizing Eq. (2), is presented in Table 6. This approach is considered the most efficient, leading to a maximum profit of $5 billion. Its validity can be confirmed by solving a Mixed Integer Linear Programming (MILP) model. As another method, the count of the workforce could be approximated to the next higher integer value. Throughout the entire planning horizon, a group comprising 67 employees produces 16,080 units, surpassing the aggregate required production by 200 units. Consequently, the possibility of terminating a worker in the second month emerges, potentially leading to a decrease of 200 units or a reduction of 5 man-months. This envisaged tactic obviates the necessity for subcontracting. Nevertheless, it resulted in a profit of $3.5 billion.

Table 6. Final summary plan.

Period (k)	Cumulative demand (D_k)	Production (P_k)	Out of source (C_k)	Inventory (I_k)	Slack (S_k)	Worker (W_k)	Number of unemployed
0	0	0	0	1141	0	92	0
1	1509	2468	0	2293	0	74	16
2	4598	2681	0	1521	0	76	0
3	7604	2579	0	989	0	73	0
4	14280	2451	38	0	1352	69	0
5	16334	2479	0	0	318	74	0
6	18017	2680	0	412	0	74	0
7	17671	4163	29	260	181	6503	177

3.3. Planning for a Multi-Product Scenario

This section is focused on strategizing for a scenario involving multiple products processed by a single unit. The demands for all products are consolidated to create a unified demand profile. As demand is fulfilled only at the conclusion of the planning horizon, this composite demand profile will display a stepwise pattern. We assume that changeover costs are roughly equivalent, and the time required for changeovers doesn't significantly vary over the planning horizon. Considering the substantial fixed production costs, each product is produced just once. Under these specified conditions and assumptions, Singh has introduced the subsequent algorithm to minimize inventory expenses (Mahdiloo et al., 2015):

- ✓ Arrange all products in ascending order based on their production rates and manufacture them accordingly.
- ✓ When dealing with products sharing identical production rates, initiate production with the one featuring a lower cost associated with holding inventory.
- ✓ In cases where products exhibit both the same production rate and inventory holding cost, prioritize the one with lower demand.

We will illustrate the application of this algorithm using a case study from the paper and paperboard industry, a classic instance of managing multiple products from a single source (Goli et al., 2020). In capital-intensive industries, the importance of maintaining elevated efficiency levels cannot be overstated. In the context of a production machine, there is a need to manufacture five distinct paper grades. Given the substantial setup expenses involved, the entire quantity of each paper grade is produced in a single production run. The inventory-related expenses associated with these high-value products typically represent a significant constraint in terms of capital flow.

The costs associated with switching between products offer advantages in terms of both profitability and electricity consumption. They also contribute to the preservation of lower-quality output at the start of a new production batch. The time required for transitioning from one product to another typically hinges on paper quality, but for the purpose of this analysis, we assume it to be nearly uniform.

Since production capacity can respond to demand, production doesn't start at time zero. Discontinuity in the production composite is due to changeover time when no production occurs. The production sequence of five products is: “newsprint”, “unlined paper”, “premium print”, “Milito” and “Lajevardi”. When considering zero changeover time, this solution is optimal in terms of cost. Yet, when we incorporate the factor of change over time, the mathematical equation results in a case study outcome within a MINLP model. This entails the maximization of profit and the identification of the production order that fulfills both the accounting demand for the overall production duration and the transition time, as dictated by the product sequence (Goli et al., 2018). The initial estimate derived from employing the algorithm significantly reduces computational time, achieving a six-fold reduction compared to the solver's default runtime. Moreover, the solution obtained using this initial estimate outperforms the solution obtained without it.

We present key criteria for establishing sequences of multiple products, as expressed in the following equations, primarily aimed at minimizing inventory expenses. When there is no changeover time, the criterion in Eq. (15) aligns with the sequence depicted in Fig. 2. When there is a non-zero changeover time, Eq. (14) offers the identical solution as outlined in the MINLP model. The production sequence becomes newspaper print, unlined paper, premium print, Lajevardi, and Milito. For instance, compared to the prior sequence, the final two products are exchanged to optimize inventory costs. This method must be expanded to encompass a broader scenario that incorporates varying changeover times and their associated costs.

Examine the inventory expenses associated with a set of products (for example, products $j1$ and $j2$) within a multi-product context where demand is exclusively met at the conclusion of the planning horizon. The production durations for each product are as follows: (Pahlevan et al., 2021):

$$t_j \equiv \left(\frac{D_j}{R_j}\right). \tag{8}$$

It is clear that two production sequences are viable: commencing with Product $j1$, followed by Product $j2$, with a changeover duration of $tc_{j1.j2}$; or initiating with Product $j2$, followed by Product $j1$, with a changeover duration of $tc_{j1.j2}$.

In the case of the initial sequence, Product $j1$ needs to be stored in inventory until the conclusion of the planning horizon following its production. Consequently, the inventory holding costs encompass the goods (starting with Product $j1$ and continuing with Product $j2$).

$$Cost_{j1} = C_{1,j}D_{j1}\left(\frac{t_{j1}}{2} + t_{j2} + tc_{j1.j2}\right), \quad (9)$$

$$Cost_{j2} = C_{1,j2}D_{j2}\frac{t_{j2}}{2}. \quad (10)$$

In the context of the second sequence, the production of Product $j2$ is initiated, necessitating its storage in inventory until the conclusion of the planning horizon. As a result, the inventory holding expenses are associated with the items, specifically those related to Product $j2$ preceding $j1$.

$$Cost_{j1} = C_{1,j1}D_{j1}\frac{t_{j1}}{2}, \quad (11)$$

$$Cost_{j2} = C_{1,j}D_{j2}\left(\frac{t_{j2}}{2} + t_{j1} + tc_{j2.j1}\right). \quad (12)$$

In the context of the primary priority, the ideal scenario is that the aggregate inventory holding cost for it (as per Eqs. (9) and (10)) should be less than the combined inventory holding cost of the items (as per Eqs. (11) and (12)) in the second sequence. Consequently, the criterion dictates the production of j1 prior to Product $j2$.

$$\frac{C_{I.j1}R_{j1}}{1 + (tc_{j2.j1}/t_{j1})} < \frac{C_{I.j2}R_{j2}}{1 + (tc_{j1.j2}/t_{j2})}. \quad (13)$$

In situations where the changeover time is uniform across all products ($tc_{j1.j2} = tc_{j2.j1}$), Eq. (13) simplifies to:

$$\frac{C_{I.j1}R_{j1}}{1 + (tc/t_{j1})} < \frac{C_{I.j2}R_{j2}}{1 + (tc/t_{j2})}. \quad (14)$$

For the case where the changeover time is zero, Eq. (13) simplifies to yield the following (Lin, 2013):

$$C_{I.j1}R_{j1} < C_{I.j2}R_{j2}. \quad (15)$$

Eqs. (14) and (15) offer straightforward and graceful guidelines for establishing the order of multiple product sequences to minimize inventory expenses, particularly when changeover times are uniform or minimal.

4. Conclusion

The expenditure breakdown analysis presented here, which utilizes a graphical approach involving production and demand composites, has demonstrated its capability to offer not only a qualitative insight into production planning challenges but also both optimal and suboptimal solutions. When striving for a chase strategy in production planning problems, this method proves to be an optimal solution by ensuring minimal inventory levels. Nevertheless, as game theory does not explicitly incorporate cost factors, it may not always ensure cost-optimized solutions in all scenarios. This principle remains applicable in scenarios involving a no-storage level strategy, in which cost implications are implicitly factored into the formulation of demand and production composites. The

optimality of the solution, whether it is optimal or suboptimal, hinges on the specific cost data and demand particulars. A similar principle applies to the stock-file scenario. When dealing with multiple products, and assuming that all product demands occur at the conclusion of the planning horizon while neglecting the effects of product changeovers (both in terms of time and cost), this approach effectively addresses the product sequencing challenge.

Game theory plays a pivotal role in attaining goals by minimizing inventory levels for a particular strategy. A hybrid approach, which leverages the strengths of game theory to provide a reliable estimate, can substantially expedite computational processes within the supply chain management (SCM) planning timeline. Game theory offers the essential adaptability necessary for swift planning, enabling managers to swiftly and precisely address inventory requirements, accept new orders, establish minimum safety stock thresholds, assess the repercussions of machine breakdowns, and gauge actual inventory expenses. In cases of minor adjustments, the impact on composites can be visually assessed without the need for complex mathematical calculations, resulting in computational time savings. This article has concentrated solely on the production stage within a centralized supply chain framework. The procurement and distribution phases have not been explicitly addressed in this analysis. In a comprehensive supply chain context, the overall planning process naturally extends to subsequent delivery stages. Future research endeavors should aim to expand upon the basic game theory approach presented herein, encompassing all relevant variables and interactions that may arise in the context of complex supply chain planning problems.

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